LECTURE 1

INTRODUCTION TO THE CONCEPT OF DIDACTIC SITUATION AND THE DISTINCTION BETWEEN SITUATIONS OF ACTION, FORMULATION AND VALIDATION

Note: All references to page numbers in brackets refer to the textbook. Other references are given in the footnotes.

1. WHAT IS A DIDACTIC SITUATION?

The notion of didactic situation is grounded in certain assumptions about learning and teaching. These assumptions can be formulated using the metaphor of game (p. 40).

* the teacher is a player faced with a system composed of a student and a didactic milieu
* the student is himself a player in a game of him/herself with a didactic milieu
* in the student's game with the didactic milieu, knowledge is the means of understanding the ground rules and strategies and later, the means of elaborating winning strategies
* the teacher's aim is to engage the student in such a game; aiming at a particular mathematical knowledge, the teacher will try to set the student-milieu system so that, indeed, this knowledge would appear as the best means available for the understanding of the rules of the game and elaborating the winning strategy

What is a milieu?

'Milieu' should perhaps be understood in an ecological sense, as in 'water is the natural milieu of fish'. Thus the 'didactic milieu is the natural milieu of students'. A person, a human being, normally lives in several different milieus and plays different roles in them. In a family milieu one can be a child, a mother, a father, etc. In a sports milieu, one can be the player, the coach, etc. Other possible roles could be played in a workplace milieu, social milieu, etc. In the school milieu, one can be a student, a teacher or an administrator. In each course, the student has to cope with a specific milieu, and there are even more specific milieus for each class in a course. To 'survive' ('to win') in a milieu one has to get to know the 'rules of the game' and develop strategies for winning the game.

Difference between this approach to teaching and learning and the traditional point of view

Learning is not reduced to the result of a transmission of information from teacher to students. Learning is understood more as sense making of situations in a milieu, and developing ways of coping with them. Teaching of a knowledge K consists in organizing the didactic milieu in such a way that knowledge K becomes necessary for the student to survive in it. If the situations in a mathematics class are such that a certain type of social behavior is sufficient for survival in them, without any use of mathematical knowledge, then it is the social behavior, not the mathematical knowledge that the students will learn. If the teacher solves the problems for the students and only asks them to reproduce the solutions, they will learn how to reproduce teacher's solutions, not how to solve problems. In this sense, the kind of game the student has to play with the milieu, to survive in it, determines the kind of knowledge that he or she will acquire. Thus, in the theory of situations, 'knowledge is [understood as] the outcome of the interactions between the student and a specific milieu organized by the teacher in the framework of a didactic situation' (Balacheff, 1993[1], p. 133).

The context of the didactic situation

A didactic situation (DS) does not exist in a void. The persons who are the main actors in the DS, playing the roles of the teacher and the student, may look at the DS from the outside, not as actors but those who plan an action in view of some far away goals. The student will see the DS as a means to reach personal life goals (e.g. becoming an engineer, obtaining a high school diploma in order to get a better paid job, etc.). The teacher will look at the DS as an educational designer, or even as a researcher, and will design the DS in view of certain professional objectives, e.g. curricular objectives, assessment objectives, or research objectives. Each person takes the existence and characteristics of the other into account in the planning.

In the diagram on page 248, the situation in which the persons who are only planning to act the roles of the teacher and the student (P1, S1) is called a 'meta-didactic situation'. When these persons act as teacher and student (P2, S2), and interact about the learning situation of the student (e.g. the teacher gives a problem to the student and the student inquires if he or she understands well the conditions of the problem) then they are in a 'didactic situation'. When the teacher withdraws from the scene, and the student engages in solving the problem for the sake of learning something, she is in a learning situation (S3). As the student endorses the problem as her own, she acts as a problem-solver (S4). In the problem, there may be a real or an imagined story with a material milieu with which some persons have to deal (S5) (e.g. persons purchasing some

[1] Balacheff, N. (1993): Artificial Intelligence and Real Teaching. In C. Keitel and K. Ruthven (Eds.), *Learning from Computers: Mathematics Education and Technology*. Berlin: Springer-Verlag (pp. 131-158)

goods and wanting to get a good deal). Sometimes the problem-solver identifies herself with these persons and solves their problem.

2. Situations of action, formulation, validation and institutionalization

There are different types of didactic situations, depending on the kind of 'game' that the teacher plays with the student-milieu system (p. 161-2). For the description of the types of situations, I decided to go from situations in which the teacher is the most authoritarian figure (the most common traditional classroom situations) to situations in which there are almost no teacher interventions.

Situation of institutionalization. The teacher plays the role of a representative of the official curriculum, the official mathematics as represented by the school institution, the textbooks officially approved by the ministry, and the official culture. He informs the students about the officially accepted terminology, definitions, theorems considered important from the institution's points of view. For the students, the milieu thus obtains the explicit features of an institution, with clear assumptions and rules. Knowledge acquires the features of a law, rather than of an answer to scientific inquiry: it is validated and justified through the authority of the institution rather than through criteria such as internal, logical consistency and relevance for the solution of scientific or technological problems.

Situation of validation. The teacher takes on the role of the theoretician evaluating the productions of other theoreticians, whose role, in the classroom, is played by the students. The students try to explain some phenomenon, or to verify a theoretical conjecture. The teacher acts as the chair of a scientific debate: s/he intervenes only to put some order in the debate among students, draw their attention to possible inconsistencies, and encourage them to be more precise and systematic in the use of concepts. For the students, the milieu resembles that of an academic seminar rather than that of a lecture room. Knowledge has the dynamic features of a theory in the making, not of a finished, institutionalized theory.

Situation of formulation. The milieu for the students is developed on the basis of some previously shared experience or activity: The students exchange and compare observations between themselves. They may not have the language to formulate their observations, so their main effort in this situation goes into creating such a language and agree on some common meanings. The teacher chairs the exchanges (in order to avoid chaos) and highlights (repeats louder, writes on the board) some formulations of the students, in case they may not have been heard by other students. Knowledge, in this situation, appears as a result of a personal experience, which needs to be communicated, and thus slightly de-personalized and de-contextualized, in order to be understood by others.

Situation of action. The teacher organizes a milieu for the students to engage with but then completely withdraws from the scene. The milieu for the students is that of a problem so chosen and formulated that (a) the students are willing to adopt it as their own, and are interested in solving it to satisfy their own curiosity or ambition; (b) the students have the means to construct the solution by themselves, either by inventing a new procedure or choosing one among those they know, without, however, the teacher suggesting which one to choose. In this situation, knowledge appears as a means for solving a problem or a class of problems.

In many mathematics classrooms, and certainly in most university lecture rooms, the institutionalization situations enjoy an absolute reign. Other kinds of situations do not appear, or they appear in degenerate forms. A degenerate form of a situation of validation is one where the students solve the 'proof' problems, e.g. 'Prove that, in a right angled triangle, if one the acute angles is 45° then the triangle is isoceles', without the statement having been formulated by the students as their own conjecture, and where the style of making the validating argument is prescribed by the teacher. A degenerate from of a situation of formulation can be one where the teacher asks the students to formulate definitions and theorems and sanctions the formulations with approval or disapproval ('correct', 'incorrect'). A degenerate form of a situation of action could be one in which the teacher gives the students a problem to solve but then constantly gives them hints and suggestions about what to do and which method to use. But scientific knowledge does not grow that way: everything starts with a problem, tentative solutions, communication of the results, their justification, revision of the results in the wake of the criticisms and queries from the scientific community. Thus the 'natural order' of the growth of scientific knowledge is from action, through formulation and validation, to institutionalization. But in school we are not inventing new knowledge; we are teaching institutionalized knowledge most of the time. So we think that it is a waste of time to reproduce, in the classroom, the tortuous path of its production, and we find that it is more economical to teach directly the results. But, in actual fact, we do not gain anything, because, by teaching only the results of scientific inquiry, we are teaching not scientific knowledge but law. So we are completely missing the point of our teaching.

Of course, we have to be realistic and admit that we cannot afford the time for teaching every single bit of the curriculum in this 'genetic' way. Therefore, we have to reflect on the knowledge we aim to teach. What parts of this knowledge can be and which cannot be sacrificed to direct institutionalization? Does it make sense to have the students re-invent decimal notation, long division rules, algebraic notation? Is it possible to create situations in the classroom through which the students, in one term, would have re-invented the differential calculus? Probably not. What is possible, then?

This is one of the questions posed by the theory of situations.

3. Example of a sequence of classroom situations: The race to 20

In order to get a better sense of the different types of situations, let us engage in a series of classroom activities that will exemplify these situations. These activities have been experimented by a team of researchers conducted by Brousseau in 1970s and described in a paper published in 1978 (pp. 4-18).

The lesson (in grade 5 or 6) was divided into several phases.

Phase 1: The teacher introduces a game to the students (5 mn)

> Teacher: *Today we'll play a game with numbers. It is called 'A race to 20'. Two players play the game. One player says '1' or '2'. The other can add 1 or 2 to the number of the opponent and says the result. He who first says '20' is the winner. Let me play this game with one of you. Anyone wants to volunteer?*

The teacher starts playing the game with a student on the board. Both she and the student write their numbers on the board. After a few steps she relinquishes her place to another student. The record of the game could be, for example:

T/S2	S1
2	3
5	7
8	9
11	13
14	16
17	19
20	

Phase 2: The students play the game in pairs (4 rounds, 10 mn)

> Teacher: *Now, sit in pairs and play up to 4 rounds of the game, keeping a record of the game on paper.*

It is expected that students will find that saying numbers at random is not the best strategy. Some will find that saying '17' is a sure guarantee of winning the game.

Phase 3: The students play the game in teams (6-8 rounds, 20 mn).

Teacher: *Now divide in two groups and play the game in teams. For each round, one student should be chosen as a representative of the team to play at the board. The teams can discuss their strategies between rounds. But do not interfere with the representative at the board.*

The teacher keeps the record of the results of the teams on the board.

Phase 4: Game of discovery: Formulation of propositions

Teacher: *Now, I am inviting each team to formulate the strategies that they think allowed them to win. The other team then verifies the statement. If the statement turns out to be true, the team wins a point. If the statement turns out to be false, the team that proved it false receives 3 points.*

If the game of discovery grinds to a halt, the teams can return to playing the game.

It is expected that the students will discover that playing 2, 5, 8, 11, 14, 17, 20 leads to winning the game, and that they will prove this statement by playing the game from each number on (action proofs).

Now, in the case of a class of MTM students, one should expect some more sophisticated proofs. For example, the students might notice that the winning numbers are of the form 20-3k, k=0, 1, 2, 3, 4, 5, 6. This could be noticed right away, or after a phase in which the teacher would propose a change of the rules of the game: adding by steps of not 1 or 2 but of 1 or 2 or 3, and racing to, say, 25. The discovery game could then be generalized: the students would be expected to formulate propositions about the winning numbers in the general case of a race to n and steps 1 through m. The students would be expected to find out and prove that

> if the race is to n and the steps are 1, 2, ..., m, (m<n) then the winning numbers are a(k) = n - (m+1)k, where k = 0, 1, 2, ..., floor(n/(m+1).

Some students could try to prove the proposition by induction, for example, like this:

Proof, by induction on k:

If k = 0 the number a(k) = n, which is a winning number by definition.

Lets assume that a(k) is a winning number and try to deduce from that that a(k+1) is also a winning number.

> If I say a(k+1) then my opponent can say any number a(k+1) + j where $1 \leq j \leq m$. But then I can add y = a(k) - a(k+1) - j, which equals a(k), a winning number, so I can win. I can add the number y because this is a number between 1 and m; proof: y = a(k) - a(k+1) - j = n - (m+1)k - n + (m+1)(k+1) - j = (m+1) - j. Since $1 \leq j \leq m$, then $-1 \geq -j \geq -m$ and $(m+1) - 1 \geq (m+1) - j \geq (m+1) - m$, i.e. $m \geq (m+1) - j \geq 1$, i.e. $1 \leq y \leq m$, and the rule of the step from 1 to m is satisfied.
> Hence, by induction, the numbers a(k) are winning numbers.

4. ANALYSIS OF THE EXAMPLE

Let us reflect on the following questions:

- What type of situation does each of the phases represent?
- What is the knowledge learned in each phase?
- What is the milieu in each phase?
- What are the rules of the game between the student and the milieu in each phase? What is the relation of the teacher to these rules?

Phase 1: In this phase we are in a didactic situation but not in a learning situation. The teacher and the students are in the roles T2 and S2, with the teacher explicitly setting up a milieu for the students and explaining to the students the rules of the game with this milieu. The milieu is an actual game - a game with numbers. The students, as students, know that they will be playing that game not just for fun but also to learn some mathematics; they do not know what mathematics they will be learning, but they promise themselves to be able to soon find out. This phase could be classified as part of a situation of institutionalization, which is spread through the whole activity, in periods when the teacher comes back and gives more instructions. The students obey the instructions because they accept the teacher's authority as a representative of the school institution: 'I don't know why I should play that game, but I trust the teacher is doing it for some purpose which is useful for me, so I'll play it'. When they think that way, the students are in the role of S3, and they see themselves in a learning situation within a didactic situation.

Phase 2: In this phase the students are in the role of S4; they are problem-solvers, coping with the game, wanting to win, and they forget, for a moment, that they are students. The milieu, in that phase, is not didactic; it is the milieu of playing games and wanting to win. The students arrive at some intuitions about the winning strategies; for example, they may find out that it is good to say '17' because they won several times by saying that number. This is a situation of action: the teacher is out of the scene, the students have endorsed the problem as their own; it is the situation itself that provides feedback and allows them to keep in control of the validity of their solutions. This way the students start developing some personal, still implicit, knowledge.

Phase 3: The students are still in the roles of problem-solvers and the milieu is still that of a game, but the situation gets slowly transformed into a situation of formulation. By having to communicate with other members of the team and thinking out some common strategies, the intuitive personal knowledge gets de-personalized and de-contextualized. There may not be much of justification at that moment, because things are happening too fast, and social rather than mathematical factors may impact on whether a suggestion of a strategy is taken or not in a team. The students may find out more winning numbers, and even describe, in general terms, their pattern (an arithmetic sequence starting from 20 with a difference of -3).

Phase 4: The students are now in the roles of S3: looking at their problem solving actions from the outside and judging them, as well as submitting them for judgment by other students. With the teacher only chairing the session, this situation classifies as a situation of validation. The knowledge developed in this phase can be that of argumentation. This argumentation may not necessarily have the features of mathematical proof. The argumentation may be of a social nature, with some students using persuasion to convince the others, rather than some kind of objective and unemotional reasoning.

The class ends in discussing the question: What kind of intervention should the teacher make in closing the activity, in a situation of institutionalization? What mathematical terminology and theory would she introduce? What was the mathematical knowledge that was aimed at in the activity?