Oral and written assignments for a course on

History of mathematics education

Program: Master in the Teaching of Mathematics
Name of the course: MATH 624L/2 AA 2012 Topics in Mathematics Education
Topic of the course: History of Mathematics Education

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Introduction

The course “History of Mathematics Education” looks at aspects of mathematics education such as

- methods of teaching as advocated by philosophers and educational reformers, and as actually practiced;
- the content of teaching;
- educational institutions and educational systems;
- values in mathematics education

in different historical periods and places:

- Ancient Greece
- Medieval Islamic civilization
- The Scientific Revolution in Europe (16th – 18th centuries)
- From the Industrial Revolution to WWI in England, USA and Quebec
- The 20th century in the USA and Quebec

There is no single textbook for this course. A variety of sources is used (books, articles, online resources).

Assessment is based on a portfolio (60%) and participation in classroom discussions (40%).

Every week, a list of readings and a number of questions or tasks related to these readings is assigned. All students are expected to prepare to discuss the readings in class. Presentation of the
readings and animation of the discussion is assured by the students and the teacher on a rotating basis.

There are eleven individual written assignments, assigned almost weekly in the course, related to the readings. Students receive feedback on their work from the teacher. A collection of the written work during the semester constitutes the core, but not the whole, of the student’s portfolio. If the portfolio contains only (revised) responses to the assigned questions or tasks, the highest grade a student can obtain is A. Only students whose portfolios will contain results of additional investigations performed from the students’ own initiative can aspire to obtain the A+ grade.

The grade depends on the clarity, thoroughness, accuracy and depth of the ideas presented in the portfolio and the quality of the student’s participation.

This file contains the readings and written assignments for the course.
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Assignment 1 – Education in Ancient Greece

Readings

A. Education in Ancient Greece in general (Lahanas, 1999)¹
B. The teaching of mathematics in Ancient Greece: (O'Connor & Robertson, 2000)²
C. The dialogue “Meno” (Plato, 380 B.C.E.)³
D. Pythagoreans’ doctrine and its influence on teaching mathematics in modern times, at (Roberts, 2008)⁴
E. Arithmetic in The Rhind Papyrus: (O'Connor & Robertson, 2000), or (Chace, 1979)
F. Number theory in Euclid’s “Elements”, book VII (Joyce, 1998)⁵

Questions about the readings

1. What are the differences and the similarities between how children and young people were educated in Ancient Greece and today?
2. Find the etymology of the word “school”. Do you think that today’s meaning of this word corresponds to its Greek roots?
3. Explain the difference between the mathematics taught in Ancient Greece to artisan classes (logistike) and that taught to upper classes (arithmetike). Illustrate your points using examples of arithmetic problems from the Rhind Papyrus and book VII of Euclid’s “Elements”.
4. Discuss the influence of the Pythagoras’ doctrine on the modern worldview and conceptions of the content of mathematics education.
5. Reconstruct the drawings that Socrates might have been making during his lesson demonstrated to Meno.
6. After reading “Meno”, how would you characterise the teaching method that Socrates is using and advocating? What is the method of reasoning employed by Socrates to help a student overcome a misconception?

¹ M. Lahanas: http://mlahanas.de/Greeks/EducationAncientGreece.htm
² J. O'Connor & M.F. Robertson: http://www-gap.dcs.st-and.ac.uk/~history/Education/greece.html
³ “Meno”: http://classics.mit.edu/Plato/meno.html
⁴ W. Roberts: http://www.principlesofnature.net/number_geometry_connections/pythagoras.htm
⁵ “The Elements”: http://www.aleph0.clarku.edu/~djoyce/java/elements/elements.html
Written Assignment 1: The Socratic Method

(a) Describe what you think are the essential features of the Socratic Method.

(b) Write a scenario of a lesson where a mathematical notion different from the one in “Meno” is taught with the Socratic Method.

(c) Explain why you think your lesson follows the Socratic Method.

References


Assignment 2 – Education in Medieval Islamic civilization

Readings

A. Mathematics in medieval Islam
B. The value of mathematics – A medieval Islamic view (Heine, 2000)
C. Methods of division transmitted from India via Arabs to Europe, especially the Galley Method (Smith, 1925/1953, pp. 136-140)
D. Check of nines: (Smith, 1925/1953, pp. 151-154)
E. Algebra according to Al-Khowârizmî and Omar Khayyam (Smith, 1925/1953, p. 382)
F. English translation of Al-Khowârizmî’s “The book of algebra and almucabola” (Karpinski, 1915). Read the following parts of the book, in this order:
   a. Introduction (pp. 67, 69; you can skip the pages with the Latin text).
   b. The list of the rules for solving the six basic types of equations presented in the book, pp. 126-127.
   c. Explanations of the rules: First Rule – Chapter III, p. 71; Second Rule – Chapter II, p. 69; Third Rule – Chapter I, p. 69; Fourth Rule – Chapter IV, p. 71; Fifth Rule – Chapter V, p. 75; Sixth Rule – Chapter VI, p. 77.
   d. Geometric demonstrations of the rules concerning the solution of the six types of equations: Demonstration of the solution of the equation $x^2 + 10x = 39$ (Fourth Rule), pp. 77-83.

Questions on the readings

1. Explain how and why the galley division method works.
2. Explain how and why the check of nines method works and what are its limitations.
3. Discuss the didactic value of the picture of a boat in understanding the galley division method. Are such means of helping students understand mathematics still used today?
4. Discuss the differences and similarities between Euclid’s geometric algebra in Book II of The Elements (Joyce, 1996) and Al-Khowarizmi’s algebra.
5. Comment on Al-Khowarizmi’s mathematical organization of algebra: What are the kinds of equations that he considered? How different is this organization from the treatment of

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7 See also [http://www.youtube.com/watch?v=uZqU_G7fRcA&feature=related](http://www.youtube.com/watch?v=uZqU_G7fRcA&feature=related)
8 Accessible online at [http://library.albany.edu/preservation/brittle_bks/khuwarizmi_robertofchester/](http://library.albany.edu/preservation/brittle_bks/khuwarizmi_robertofchester/)
quadratic equations in present day College Algebra textbooks (see, for example, (Blitzer, 2004) or any other College Algebra textbook in the library)? What similarities do you see?

**Written Assignment 2: Algebra at the time of Al-Khowarizmi**

Comment on Al-Khowarizmi’s mathematical organization of algebra: What are the kinds of equations that he considered? How different is this organization from the treatment of quadratic equations in present day College Algebra textbooks (see, for example, (Blitzer, 2004) or any other College Algebra textbook in the library)? What similarities do you see?

**References**


Assignment 3 – The 16th century in Europe and the catechetic method of teaching

The catechetic method of teaching is commonly described as “consisting in asking questions and receiving answers”. The name “catechetic” comes from the question-and-answer form in which the Catechism of the Catholic Church (CCC) has often been put, especially for the purposes of teaching it to children. Example from CCC:

**Question 204** Why are the Creeds called 'symbols of Faith'?
*Answer:* The Creeds are called 'symbols of Faith' because 'symbolon' in Greek means 1. A summary; 2. One half of a broken object presented as a token of recognition.

**Question 2204**: To what virtue does it belong to respect the name of God?
*Answer:* It belongs to the virtue of religion to respect the name of the Lord.

Some philosophers of education had a more narrow definition of the catechetic method. It used to be likened to or distinguished from other methods such as the dogmatic method, the Socratic Method, or the dialogic method. We can learn about such analogies and distinctions using the readings listed below.

**Readings**

A. In his catalogue of old American arithmetic books, the historian and educator David Eugene Smith (1860-1944) described Jodocus Willichius’ Arithmeticae (1540) and Robert Recorde’s “The Whetstone of Witte” (1557) as having adopted a catechetical form. Read Smith’s account of Willichius’ Arithmeticae and see a reproduction of a page from Willichius’ book (Smith E. D., 1908/2007, pp. 197-198).

B. Robert Recorde’s “The Whetstone of Witte” (1557) is written as a conversation between two people, one called “master” and the other – "scholar". See how the master introduces the scholar to variables, which he calls ‘cosike numbers’.

C. Galileo Galilei’s “Dialogues on the two chief world systems, the Ptolemaic and the Copernican” (Galilei, 1967).

D. Immanuel Kant’s distinctions between different methods of teaching (Kant, 1904, pp. 182-185, 260-262, 280-281).

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F. Arithmetic presented in a 19th century textbook for teachers in Quebec (Valade, 1851, pp. 164-186)12. (in French)

Questions on the readings

1. How would you qualify the method of teaching presented in Valade’s textbook for elementary school instructors?

2. Some sources equate the Catechetic method of teaching with the Socratic method, e.g. The free online dictionary13. Kant considers them as distinct methods. Present Kant’s view and the distinctions among teaching methods that he makes. What is your opinion on this matter?


4. How would you qualify the form used by Galileo in “Dialogues on the two chief world systems, the Ptolemaic and the Copernican”: Dogmatic? Catechetic? Socratic? Dialogic? Or other?

5. In communicating mathematics examples are often used.
   a. Can one teach mathematics using the Catechetic method, never use examples and yet have all students succeed in the course?
   b. It seems to matter what examples we use to communicate the meaning of a mathematical idea. Kant distinguished two kinds of examples, one called, in German, Beispiel and the other – Exempel (Kant, 1904, pp. 280-281). Explain the difference between these two kinds of examples.
   c. How would you qualify the examples used in Rhind Papyrus? Beispiel? Exempel? Or yet another category?

6. What does Recorde mean by “cosike numbers”?

7. Diagrammatic representations of numbers underlie Recorde’s distinctions of kinds of numbers: “flatte numbers”, “square numbers”, “cubic numbers”. Such geometric imagery has its sources in Pythagorean arithmetic and the algebraic geometry presented in Euclid’s

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12 F.-X. Valade: http://openlibrary.org/books/OL24771349M/Guide_de_l%27instituteur

“Elements” book II (Joyce, Euclid’s Elements, 1996). Draw diagrams that could represent Euclid’s Propositions II.1, II.2, II.3 and II.4.

8. Present and critique Tonstall’s explanation of the multiplication of fractions.

9. Present and explain the “proof by casting out the nines” described in Valade’s guide for teachers.

Written Assignment 3: Different kinds of examples. Recorde’s Algebra

1. Illustrate Kant's distinction between two kinds of examples, Beispiel and Exempel using examples from mathematics.

2. Find a couple of examples of a present-day introduction to the notion of variable in secondary schools or colleges. How different are they from Recorde's introduction to “cosike numbers”?

References


Recorde, R. (1557). The whetstone of witte.


Assignment 4 – The 17th century in Europe: New pedagogical ideas and the tradition of the Rule Method in the teaching of Arithmetic

Readings

A. The pedagogy advocated by Johann Amos Comenius (1592-1670) – Moravia
   a. Comenius’ original work: *Orbis sensualium pictus* (Comenius, 1968), especially the page on Geometry (pp. 208-209)
   b. Commentaries on Comenius’ ideas and their impact:
      - James Bowen, Introduction to “Orbis sensualium pictus” (Comenius, 1968, pp. 1-33)
      - A short article on Comenius at StateUniversity.com website

B. The pedagogy of Jean-Baptiste de la Salle (1651 – 1719) – France
   a. Wikipedia article
   b. Sections, “La méthode pédagogique”; “La méthode des Frères des Écoles Chrétienes” in (Charland, 2005, pp. 20-22)
   c. Article by Houssaye & Dancel (2007) (in French)

C. The teaching of practical arithmetic in England; a textbook by Edmund Wingate (1596-1656)
   a. Wingate’s life in brief
   b. Teaching of the use of proportions in solving arithmetic problems in a 19th century edited version of “Wingate’s Arithmetick”:
      - Rule of Three and Rule of False (Wingate, 1713, pp. 69-102, 125-132);
      - “Demonstration” of the Rule of Three and Rule of False in the Appendix.
   c. Teaching of arithmetic before Wingate: Robert Recorde’s approach (Cajori, 1896, pp. 183-185)

D. The Rule Method vs the Analytic Method, explained on the example of 19th century texts (Michalowicz & Howard, 2003)

18 “Mr. Wingate’s Arithmetick”: [http://books.google.ca/books/about/Mr_Wingate_s_Arithmetick.html?id=Vq82AAAMAAJ&redir_esc=y](http://books.google.ca/books/about/Mr_Wingate_s_Arithmetick.html?id=Vq82AAAMAAJ&redir_esc=y)
19 F. Cajori: [http://openlibrary.org/books/OL6595572M/A_history_of_elementary_mathematics](http://openlibrary.org/books/OL6595572M/A_history_of_elementary_mathematics)
Questions on the readings

1. What were the main principles of Comenius’ pedagogy and what was innovative in it?
2. In Orbis sensualium pictus, there is one plate directly related to mathematics: “Geometry” (Comenius, 1968, pp. 208-9). What does it say about geometry? What are the instruments represented on the picture? Identify the mathematical laws based on which these instruments work.
3. De la Salle’s method of teaching is characterized as: simultaneous pedagogy and differentiated pedagogy. Explain what this meant in practice.
4. Describe the Rule and the Analytic methods of teaching arithmetic.
5. Would you agree with the opinion that Robert Recorde’s method of teaching arithmetic was closer to the Analytical method than to the Rule method? Why yes or why not?
6. Show, on examples, how the different Rules of Three (direct, inverse, simple, compound) were used to solve problems in Wingate’s Arithmetick. How would you solve such problems today?
7. Show, on examples, how the Rule of False Position works.

Written Assignment 4: Rules of Three and Rules of False Position

How would you teach (a) the Rule of Three Inverse; (b) the Rule of False, using the Analytic method?

References


20 F.-X. Valade: http://openlibrary.org/books/OL24771349M/Guide_de_l%27instituteur


Assignment 5 – The 17th century in Europe: The mathematical education of Isaac Newton. Newton’s communication style

Readings

B. Some of the many books that Newton studied:
   b. An “arithmetic” approach to solving geometric problems combined with an inductive method of both investigation in mathematics and communication of the results: John Wallis’ Arithmetica Infinitorum (Stedall, 2001); (Wallis, 1656)22
   c. René Descartes’ mathematical work:
      i. “La géométrie”23, excerpts in English in (Fauvel & Gray, 1987, pp. 336-340)
      ii. Descartes’ method of generating curves using mechanical devices (Cooke, 2005)
C. Newton’s style of investigation and communication in mathematics based on excerpts from his writings on “fluents” (functions), “quadratures” (integrals) and “fluxions” (derivatives) in (Fauvel & Gray, 1987)

Questions on the readings

1. Newton’s education:
   a. What did Newton study in the Grantham grammar school? Was anything he learned useful for his future studies?
   b. What did Newton read outside of that school?
   c. What were Newton’s hobbies during his adolescent years?
   d. What was Newton expected to study at Cambridge?
   e. What did Newton actually read and study as a student at Cambridge?
   f. What do Newton’s notes in the form of “Quaestiones” say about his approach to his readings?

21 J. Bates: http://archive.org/details/mysteryesofnatur00bate
22 J. Wallis: http://books.google.ca/books?id=iBaDonchWNcC&printsec=frontcover&source=gbs_ge_summary_r&cad=0#v=onepage&q&f=false
g. Characterize Newton’s method of studying science.

2. Wallis qualified his method of mathematical investigation as “induction”. He also used it in presenting mathematical ideas and results. Newton seems to have adopted this method in his investigations and, to a certain extent, in communicating his results. Describe this method.

3. Descartes’ mathematics
   a. Explain how Descartes’ method for constructing the square root of a number using a ruler and compasses works and why it works.
   b. Describe Descartes’ method of finding “unknown lines” (p. 3-5 of La géométrie).

4. If possible, use the method of indivisibles that Newton demonstrates in the excerpt titled “Quadrature as the inverse of fluxions” to solve the following problem, and comment on the results of your experience:
   Given the area \( z \) under a curve \( y \) in terms of \( x \), find the equation of \( y \) in terms of \( x \), for
   a. \( z = \sin(x) \)
   b. \( z = x - \frac{1}{6} x^3 \)

**Written Assignment 5: Inductive method. Mechanical curve drawing devices.**

**The method of infinitely small quantities.**

1. Write a plan of a lesson where the teacher would be using the inductive method to introduce students to a mathematical topic.
2. Explain how Descartes’ linkage device presented in Cooke (2005, p. 353) works.
   Simulate the functioning of the linkage with Geogebra.
3. Write an introductory lecture on limits for college students (in a one-variable Calculus course), borrowing ideas from Newton’s way of communicating and using this notion.

**References**


Assignment 6 – Pedagogical reformers of the late 18th and beginning 19th century: Pestalozzi, Herbart and Lancaster

Readings

A. The life and times of Johann Heinrich Pestalozzi (1746-1827), based mainly on Green’s account of “The educational ideas of Pestalozzi”\(^ {24}\) (Green, 1900).
   a. The historical and cultural context: Between Enlightenment and Romanticism: Green, Chapter I: General historical introduction (Green, 1900, pp. 1-15).
   b. Urbanization and industrialisation: (Button & Provenzo, Jr., 1989, pp. 70-72)
   c. Chapters II and III: Pestalozzi’s life (Green, 1900, pp. 16-68)

B. Pestalozzi’s pedagogical ideas
   a. Aims of education: Chapters IV. The aim of education; V. Intellectual Education; VI Practical education (Green, 1900, pp. 69-150)
   b. On teaching in the context of work: (Pestalozzi, 1894, p. xviii)
   c. Teacher training: Chapter VIII and parts of Chapter IX (Green, 1900, pp. 151-164; 173-176)

C. The teaching of arithmetic according to Pestalozzi:
   a. On teaching of arithmetic in general: (Green, 1900, pp. 179-181); (Pestalozzi, 1894, pp. 132-138)
   b. On teaching the notion of triangle: (Pestalozzi, 1894, p. 131)
   c. On teaching measurement and proportion: (Pestalozzi, 1894, pp. 116-122)
   d. On teaching the notion of number:
      i. Number as ratio: (Pestalozzi, 1894, p. 132)
      ii. Numbers as abstractions from magnitudes (Pestalozzi, 1894, pp. 207-209)
      iii. On teaching fractions: (Pestalozzi, 1894, pp. 237-9)

D. Friedrich Herbart (1776-1841): pedagogy as a scientific method
   a. Herbart as a follower and constructive critic of Pestalozzi: (Green, 1900, pp. 169-171); (De Garmo, 1895, pp. 3-11)
   b. The pedagogical ideas of Herbart: (De Garmo, 1895)\(^ {25}\)
      i. Aims of education: (De Garmo, 1895, pp. 47-56)
      ii. Motivation: (De Garmo, 1895, pp. 57-66; 75-77)

\(^ {24}\) J.A. Green: [http://openlibrary.org/books/OL14005787M/The_educational_ideas_of_Pestalozzi](http://openlibrary.org/books/OL14005787M/The_educational_ideas_of_Pestalozzi)

\(^ {25}\) C. De Garmo: [http://archive.org/details/herbartherbartia00degauoft](http://archive.org/details/herbartherbartia00degauoft)
iii. Teaching methods: (De Garmo, 1895, pp. 67-82); three methods: presentative, analytic, and synthetic (De Garmo, 1895, pp. 71-73)

iv. Herbart's negative opinion about teaching religion in schools (De Garmo, 1895, p. 73)

v. School administration: (De Garmo, 1895, pp. 83-100)

  c. The educational ideal based on Herbart's philosophy of education: (De Garmo, 1895, pp. 228-256)

E. Lancaster's monitorial system of education: (Button & Provenzo, Jr., 1989, pp. 75-78)

Questions on the readings

1. According to Pestalozzi, good education must foster the development of the "heart, the head and the hand, all at the same time, i.e., it must develop reasoning while building character and training manual skills. Sketch a mathematics lesson where this principle is put into practice.

2. Explain the difference Herbart makes between two forms of maintaining school discipline: by government, and by training. Give examples of each in the context of mathematics teaching. Is there an analogy between this distinction and that between extrinsic and intrinsic motivation?

3. Describe the effect of urbanization and industrialisation on the living conditions and educational reforms at the turn of the 19th century.

Written Assignment 6: A Herbartian lesson

Design a "Herbartian" lesson on a chosen mathematical topic. The lesson should have the following characteristics:

  c. The instruction is synthetic (rather than presentative or analytic).
  d. The teacher avoids forcing voluntary attention in students by remote means such as good marks or competition.
  e. The lesson creates favourable conditions for apperceiving attention in students.
  f. The lesson creates favourable conditions for alternating states of mental absorption in a task and reflection on the results of the work on this task.
  g. The lesson is organized into four phases:
     I. (Clearness) The teacher presents a task and students become mentally absorbed in it.
     II. (Association) Progress, through reflection, from one absorption to another, fuelled by a conversation between the teacher and the students in a way that gives the students opportunities to investigate the task, to change previous approaches to its solution, and to assimilate new knowledge.
III. (System) Synthesis of the results of the work in phase II in a connected and coherent discourse, through a “rich arrangement of a rich reflection” to “bring about the highest scientific organization of which the pupil is capable”, by way of a conversation between the teacher and the students (De Garmo, 1895, pp. 80-1).

IV. (Method) Students work individually on tasks prepared by the teacher; the tasks are meant to foster progressive reflection.

References


Green, J. (1900). *The educational ideas of Pestalozzi.* Baltimore, MD: Warwick & York, Inc.

Assignment 7 – Teaching mathematics in the 19th century USA

Readings

A. Elementary schools in the 19th century USA: (Button & Provenzo, Jr., 1989, pp. 79-85)
B. Arithmetic teaching in the 19th century USA (Cajori, 1890, pp. 9-18; 45-55)26
C. The 19th century arithmetic textbooks:
   a. Thomas Dilworth’s “The Schoolmaster’s Assistant” (Dilworth, 1825)27
   b. Nathan Daboll’s “The Schoolmaster’s Assistant”28: Introduction to fractions (Daboll, 1820, pp. 74-77)
   c. Overcoming the Rule Method by teaching mathematics according to Pestalozzi: Warren Colburn’s “First lessons in Arithmetic upon the plan of Pestalozzi, with some improvements”:
      i. The original version from 1825: Introduction and questions on fractions (Colburn, 1825, pp. iii-xii; 45-50)
      ii. A revised version from 1884: The initial story and Sections VIII and IX on Fractions (Colburn, 1884, pp. 99-122)29
      iii. An article analyzing Colburn’s textbook: (Monroe, 1913)
   d. Overcoming the Rule Method by teaching Demonstrative Arithmetic: Augustus de Morgan, 1840, Elements of Arithmetic (Morgan, 1840)30
      i. De Morgan's philosophy of teaching arithmetic, Preface, pp. iii-viii
      ii. De Morgan's explanation of the notion of number, Book I, Section I, paragraphs 1 and 2, pages 1-2.
      iii. De Morgan’s introduction to the signs used in arithmetic: +, -, x, division sign and equality sign: Book I, Section I, paragraph 23, p. 10-11

26 F. Cajori: http://openlibrary.org/books/OL13506524M/The_teaching_and_history_of_mathematics_in_the_United_States
27 T. Dilworth: http://archive.org/details/schoolmastersas02dilwgoog
28 N. Daboll: http://openlibrary.org/books/OL7122302M/Daboll's_Schoolmaster's_assistant_improved_and_enlarged
29 The 1884 edition of Colburn’s arithmetic textbook can be read online at http://openlibrary.org/works/OL6726542W/Warren_Colburn%27s_First_lessons

D. Teaching mathematics at the college level in the early 19th century in the US:
   a. General description: (Cajori, 1890, pp. 55-56)
   b. A textbook for teaching mathematics at the University at Cambridge (Massachusetts): (Webber, 1801)

E. The mid-19th century educational reforms in the US particularly the “Common Schools Movement”:
   a. The rise and growth of the Common Schools Movement in the 19th century USA: (Button & Provenzo, Jr., 1989, pp. 93-148)
   b. The voice of one of the main reformers: Horace Mann’s 12th Report: (Mann, 1957, pp. 79-112)
   c. Critique of the reforms:
      i. “The Irony of Early School Reform”: The social context of the reforms: Urbanization and industrialisation; Study of cases of imposition of and resistance to a new school system (Katz, 1968, pp. 1-114; 213-218);
      ii. “Class, Bureaucracy and Schools”: Introduction: between the ideal and the reality of schools; Relationships between bureaucracy and class bias; Suggestions for reformers (Katz, 1975, pp. xv-xxiii; 108-113; 140-146)

Questions on the readings

1. About Dilworth’s “The Schoolmaster’s Assistant”:
   a. What methods of teaching does Dilworth use?
   b. What is your opinion on his definition of fraction?
   c. Today, fractions in elementary schools are taught in the context of everyday life situations. Dilworth says that he is teaching both "practical and theoretical arithmetic". Is his way of giving meaning to fractions similar to today's ways?
   d. Is Daboll's “The Schoolmaster's Assistant” any different from Dilworth's? If yes, how?

31 Cajori's book on the history of mathematics teaching in the US can be read online at: http://openlibrary.org/books/OL13506524M/The_teaching_and_history_of_mathematics_in_the_United_States
32 The book can be read online at http://openlibrary.org/books/OL6930841M/Mathematics
33 Some examples of how fractions are taught today are described at http://www.superteacherworksheets.com/fraction-cont.html
2. About Webber's "Mathematics":
   a. Compare the content of Webber's "Mathematics" with the mathematics curriculum taught at present-day universities, for example, at Concordia.
   b. What differences and similarities do you see between Webber’s approach to teaching Algebra (Webber, 1801, pp. 263-380) and present-day North American College Algebra textbooks?

3. Questions on Colburn's “First lessons” and lessons on fractions (Colburn, 1825).
   a. Explain how Colburn understood teaching arithmetic in general and fractions in particular, “on the plan of Pestalozzi”.
   b. In what way can you say that Colburn’s approach was based “on the plan of Pestalozzi”?
   c. Describe and critique the way of teaching fractions in the 1884 edition of Colburn's textbook (Colburn, 1884, pp. 99-122).

4. Questions about De Morgan’s “The Elements of Arithmetic”:
   a. Compare De Morgan’s approach to the notion of number and arithmetic operations to the approach to the same notions in F.-X. Valade (Valade, 1851).
   b. Compare De Morgan’s and Colburn's approach to the teaching of fractions.

5. Questions about the mid-19th century school reforms in the US:
   a. What was the social context of the reforms?
   b. What were the expected outcomes of the reforms?
   c. Who supported and who was against the reforms?
   d. Some authors claim that the reforms were based on certain myths. What were those myths?
   e. What could have been the reasons of the (relative) failure of the reforms?
   f. What aspects of the school system introduced in the 19th century reforms in the US have remained until today? Which of them you think are fortunate and which are less so?
   g. Do you think Katz' suggestions for the reformers are useful? How realistic do you think they are?
Written Assignment 7: Children’s difficulties in arithmetic according to Colburn and De Morgan


References


Webber, S. (1801). Mathematics: Compiled from the best authors and intended to be the textbook of the course of private lectures on these sciences at the university at Cambridge. Boston: Thomas Andrews.
Assignment 8 – Education in Quebec from the British Conquest in 1763 to 1900

Readings

A. A brief history of education in Quebec from 1763 to 1900 (in English): (Magnuson, 1980, pp. 11-67)

B. History of the 19th century Quebec education seen by a Quebecois (in French) (Charland, 2005):
   a. Schooling in the Province of Quebec at the turn of the 19th century (Charland, 2005, pp. 51-52)
   b. Liberal ideals of education in mid-19th century Quebec: Pierre-Martial Bardy et Jean-Baptiste Meilleur (Charland, 2005, pp. 52-55)
   d. The functioning of the Examiners’ Boards (Charland, 2005, pp. 71-72)

C. The contents of elementary school teacher’s education – (in French) (Charland, 2005, pp. 71-72; 82-85); (Valade, 1851, pp. 164-258); (Langevin, 1869, pp. 5-19; 24-27; 38; 46-60; 95-101)

D. In 19th century Quebec, textbooks were often imported from France. Look at one 19th century French textbook for secondary schools (in French): “Eléments d’Algèbre et de trigonométrie” (Lusson & Courcelles, 1872, pp. 35-37).

Questions on the readings

1. Identify the sources and the consequences of opting for denominational school boards in the 19th century Quebec.

2. What is the difference between Civil Law and Common Law traditions? What can be the impact on education of the adoption by a state of the Civil Law system, as opposed to the Common Law system?

3. Magnuson mentions (1980, pp. 51-52) that French Canada had a long tradition of trade and technical education dating from the late seventeenth century. Find evidence to support this claim. Has this tradition been maintained today?

34 F.-X. Valade: [http://openlibrary.org/books/OL24771349M/Guide_de_l%27instituteur](http://openlibrary.org/books/OL24771349M/Guide_de_l%27instituteur)
4. Questions on the *Cours de pédagogie* by Jean Langevin:
   a. What difficulties does an elementary school teacher have to face, according to Jean-Langevin? (Langevin, 1869, p. 5)
   b. Do you think you satisfy Langevin’s conditions of becoming a dedicated teacher? (Langevin, 1869, pp. 6-7)
   c. Which of the moral qualities of the teacher mentioned by Langevin (Langevin, 1869, pp. 7-15) are still expected of teachers today?
   d. What is education and how different is it from instruction? (Langevin, 1869, pp. 16-19; 38)
   e. What is intellectual education, according to (Langevin, 1869, pp. 24-27)?
   f. Do you agree with Langevin’s description of the qualities that good teaching should possess? (Langevin, 1869, pp. 46-54)
   g. Describe, in your own words, the teaching methods that Langevin describes (Langevin, 1869, pp. 55-60)
   h. What was a future teacher aspiring to obtain an “academic certificate” expected to know in mathematics according to Langevin (Langevin, 1869, pp. 95-101), and according to Valade (Valade, 1851, pp. 164-258)?

5. In the textbook by Lusson & Courcelles, 1872, the algebraic identity related to the difference of squares is formulated in words as “la différence des deux carrés est égale à la somme de leurs racines carrées multipliée par la différence de ces mêmes racines” (the difference of two squares is equal to the sum of their square roots times the difference of those same roots). Is the verbal representation saying exactly the same thing as, \(a^2 - b^2 = (a - b)(a + b)\)? Are the two representations algebraically equivalent?

6. How different is the algebra presented in Lusson & Courcelles from its presentations by Al-Khowarizmi, Robert Recorde, and present-day College Algebra textbooks?

**Written Assignment 8: The art of interrogation**

Apply one of Langevin’s principles of the art of interrogation (Langevin, 1869, pp. 52-54), namely the principle of using multiple versions of the same question during a single interrogation period, in planning a sequence of questions on the idea of exponential function in a College Algebra class.
References


Assignment 9 – Educational ideas in the early twentieth-century US.
The importance of intuition and applications in mathematics education

Readings

A. John Dewey “The child and the curriculum” (Dewey, 1902)
B. E. Walmsley’s brief history of mathematics education during the 20th century (Walmsley, 2007): sections pertaining to the period 1900-1950.
C. David E. Smith, “The teaching of Arithmetic” (Smith D. E., 1909); (Donoghue, 2007). In Smith’s book, read, in particular: Chapter VI: Method (pp. 22-25); Chapter IX: Children’s analyses (pp. 35-38); Chapter XII: Certain great principles of teaching arithmetic (pp. 52-54).
E. E.H. Moore’s laboratory method of mathematics teaching: (Donoghue, 2003, pp. 338-340); (Roberts, 2001); (Moore, 1967); (Myers, 1909a) (Myers, 1909b).

Questions on the readings

1. Comment on J. Dewey’s metaphor about the “old education” as “forever tasting, never eating”. Reflect on the negative consequences this kind of education may have for students’ experience with mathematics (Dewey, 1902, pp. 16-17).
2. What does Dewey’s postulate to “reinstate into experience the subject-matter of the studies” (Dewey, 1902, pp. 22-3) mean for the teaching of mathematics; what would the educators, both researchers and teachers, have to do to teach mathematics well, according to Dewey?
3. State D.E. Smith’s principles of teaching arithmetic.
4. In what sense has D. E. Smith been a “pioneer”?

36 J. Dewey: http://archive.org/details/childandcurricul00deweuoft
38 Myers’ “First-year mathematics for secondary schools” can be read online at: http://archive.org/details/firstyearmathem00myeriala
39 Myers’ “Teacher’s manual for First-year Mathematics” can be read online at: http://archive.org/details/teachersmanualf00myergoog
5. In “Anne of Green Gables”, a famous Canadian novel published in 1908, there is a passage about the teaching and learning of geometry. In this part of the book, Anne is preparing her lessons for the next day in the Green Gables kitchen. Matthew is also in the kitchen, dozing over a copy of “Farmer’s Advocate”. At some point, Anne starts a conversation with Matthew, reproduced below.

- ‘Matthew, did you ever study geometry when you went to school?’
- ‘Well, now, no, I didn’t,’ said Matthew, coming out of his doze with a start.
- ‘I wish you had,’ sighed Anne, ‘because then you’d be able to sympathize with me. You can’t sympathize properly if you’ve never studied it. It is casting a cloud over my whole life. I’m such a dunce at it, Matthew.’
- ‘Well, now, I dunno,’ said Matthew soothingly. ‘I guess you are all right at anything….’
- ‘I’m sure I’d get on better with geometry if only [the teacher] didn’t change the letters,’ complained Anne. ‘I learn the proposition off by heart, and then he draws in on the blackboard and puts different letters from what are in the book and I get all mixed up. I don’t think a teacher should take such a mean advantage, do you?…’

(a) What can you infer from this passage about what it meant to teach and learn geometry in Lucy Maud Montgomery’s (the author’s of the book) times?
(b) In what ways did Wentworth’s “Plane geometry” depart from this approach? Justify your answer using quotes from Wentworth’s book. You may also refer to (Donoghue, 2003, pp. 335-337).

6. Study the chapter on the Pythagorean Theorem in Wentworth (Wentworth, 1899, pp. 238-241) from the perspective of a teacher or a student in a course conducted according to Wentworth’s method. This method is described in Wentworth’s “Note to Teachers” (Wentworth, 1899, pp. v- vi) and in Donoghue’s chapter (Donoghue, 2003, p. 336). Be prepared to play the role of the student or the teacher in a class on the Pythagorean Theorem conducted according to Wentworth’s method. The roles (teacher or student) will be distributed in class.

7. Construct, with a ruler and compasses, a regular pentagon. Then construct, also with ruler and compasses, a triangle whose area is equal to that of the regular pentagon.

8. State the main postulates of E. H. Moore’s vision of mathematics education.

9. Moore refers to Henri Poincaré and Felix Klein. Who were these people? What did they do for mathematics education?
10. Donoghue (Donoghue, 2003, p. 338) claims that Myer’s “First-Year Mathematics for Secondary Schools” (Myers, 1909a) was an attempt to realise Moore’s vision of mathematics education in secondary schools. Identify those aspects of this vision that are reflected in Myers’ treatment of geometry in the textbook in Chapters VII and XIV. Support your claims with quotes from the textbook.

11. How useful do you think would be Myers’ “Teacher’s Manual for First-Year Mathematics” for you, were you to teach a course with his textbook?

Written Assignment 9: Moore’s method

Sketch a sequence of three lessons on (a) the Pythagorean Theorem, (b) a mathematical topic of your own choice, using E.H. Moore’s “laboratory method”.

References

Assignment 10 - Mathematics education reforms in the 1960s: 
New Math, Modern Mathematics, La mathématique moderne

Readings

A. The intellectual context of the New Math movement
   a. Structuralism as a prevailing philosophical perspective in many domains: linguistics, 
      semiotics, anthropology, psychology, architecture, and mathematics, among 
      others.⁴⁰
   b. Jean Piaget:
      i. Structuralism in cognitive psychology
      ii. Theory of developmental stages (Sinclair, 1971)
   c. In mathematical research,
      i. expansion of the domain of “Foundations of Mathematics” focused on the 
         methodology of mathematics and characteristics of mathematical theories; 
         development of “meta-mathematics”, i.e., a theory of mathematical theories 
         which includes, in particular, definitions of truth, definition, theorem and 
         proof and attempts to answer questions about consistency, completeness 
         and categoricity of mathematical theories.
      ii. reformulation of mathematical theories in the languages of logic, set theory 
          and algebraic structures (groups, rings, fields, modules, vector spaces, 
          algebras, etc.), undertaken, in particular by the Bourbaki group⁴¹;
          Construction of a meta-theory of algebraic structures: category theory

⁴⁰ Common to all structuralisms is the conceptualization of phenomena in terms of closed (self-contained) 
systems governed by some inner rationality, so that the functioning of the system can be explained by the 
laws of this rationality, without drawing on elements external to it. Such systems are called “structures”. Since 
structuralisms usually aimed at modeling human phenomena (e.g., language, cognition, rites and rituals, etc.), 
they focused not so much on static elements but on their transformations and relations among them. Thus, 
Piaget’s first approximation of a definition is that a structure is a closed system of transformations whose 
interactions are governed by certain laws; closure means that anything produced by the interplay of the 
transformations is again absorbed by the system. In mathematics, a structuralist perspective means, in 
particular, viewing numbers not in their relation to objects outside of mathematics (e.g., as measures of some 
of their aspects), but in their relations with each other within closed systems (algebraic structures). (based 
on Piaget, J. 1970, Structuralism)

⁴¹ Read the online article about Bourbaki at: http://planetmath.org/NicolasBourbaki.html
B. The political context of the New Math movement
   a. In the USA: (Garrett & Davis jr, 2003, pp. 512-515)
   b. As a global phenomenon: (Moon, 1986, pp. 43-69)

C. The main tenets of the New Math movement
   a. In the USA:
      i. From the New Math to the Agenda for Action: (Fey & Graeber, 2003, pp. 521-527)
   b. In Europe:
      i. Proceedings of the 1963 OECD conference on New Math in Athens (Fehr, 1963, pp. 3-73)

D. Examples of “modern” mathematics textbooks and materials
   a. SMSG textbook for a first course in algebra: (Peters & Schaaf, 1963, pp. v-viii; 348-349; 376-380; 395-6) (Pythagoras)
   b. Radical modernism: George Papy’s Modern Mathematics (Papy, 1968, pp. v-xvii; 440-459)
   c. Zoltan P. Dienes’ activities (Dienes, 2002)
   d. SMP textbooks: for students (SMP, 1967, pp. 78-81; 195-209); for teachers (SMP, 1970, pp. 166-182)
   e. Textbooks for pre- or in-service elementary school teachers education: (Kenyon, 1969, pp. v-xii; 205-222; 273-278)
   f. An axiomatic approach to arithmetic: (Moise, 1966)
   g. Popularization of the New Math: W. W. Sawyer, A path to modern mathematics, (Sawyer, 1971, pp. 24-26) (informal introduction to Linear Algebra; application of vector calculus to prove classical theorems in geometry).

E. Reactions to the New Math movement
   a. Bob Davis: (Davis, 2003)
   b. Caleb Gattegno: (Gattegno, 1963, pp. 1-4; 32-60)
   c. Hans Freudenthal (Freudenthal, 1969)

Questions and Tasks about the New Math movement

1. What are the stages of development, according to Piaget?
2. Explain how Piaget’s theory was influenced by the idea of algebraic structure in mathematics.

3. What was the problem that triggered the start of the Bourbaki group?

4. What was the political context of the New Math movement in the US?

5. Solve Ex. 1 (Show that $2 + 3 = 5$), and Ex. 7 (Show that $a + a = 2a$) from the textbook “The numbers systems of elementary mathematics” (Moise, 1966). What do you think about the way arithmetic operations are introduced in this textbook?


7. Do you agree with Papy’s claims, presented in the Preface to (Papy, 1968) that
   a. “Previously, elementary mathematics teaching could only deal with artificial situations in which pieces of technical work were mixed in with vague (and usually not explicit) appeals to intuition”?
   b. his textbook encourages children to “take an active part in the building up of the mathematical edifice starting from simple, familiar situations”?

8. What were Gattegno’s views on teaching mathematics? In what ways did they depart from the New Math tenets and in what ways were they close to them? Base your response on (Gattegno, 1963, pp. 49-50; 55-60).

9. What is Freudenthal’s point in the following fragment (Freudenthal, 1969, p. 38):

   7a. I would add a warning against one type of problems which is quite common in set theory and which betrays the poor didactics of this new school instruction subject. First a digression to teaching arithmetic: There is a type of problems “John has five marbles, he gets three more, how many marbles does he now have?” To solve them, the pupil must know more than adding five and three. He must know which operation he has to perform under the given circumstances. This ability, which is exercised by this kind of problems, guarantees the applicability of arithmetic.
   
   Now a problem on sets: $A$ is the set of adults, $B$ is the set of males, what is $A \cap B$?
   
   Compare this with the arithmetical problem. Are not you shocked? What happens here, would correspond to an arithmetical problem of the kind:
   
   John has five marbles, he gets three more, add five and three.
   
   Is not this too bad?
Written Assignment 10: The Pythagorean Theorem in the New Math era

Discuss the merits and shortage of the presentations of the Pythagorean Theorem in the texts listed below. In what ways is each of them similar and in what ways is it different from the presentation of the Pythagorean Theorem in Wentworth’s “Plane Geometry”?

   a. Modern Elementary Mathematics (Kenyon, 1969)
   b. The School Mathematics Project Book 3 and Teacher’s Guide, Chapter 12 “Shearing” (SMP, 1967); (SMP, 1970)
   c. Gattegno’s “Note on Pythagoras’ theorem” (Gattegno, 1963, pp. 32-38)

References


Assignment 11 – Educational ideas in Quebec and English Canada from 1900 until today

Readings

A. Mathematics education in English Canada: 1900-1950: (Sigurdson, Kieren, Pothier, & Roulet, 2003, pp. 195-238)
C. History of education in Quebec – The Quiet Revolution:
   a. The general historical context: (Magnuson, 1980, pp. 102-106)
   b. The Parent Commission recommendations and their effects: (Magnuson, A brief history of Quebec education: From New France to Parti Québécois, 1980, pp. 106-123);
D. Mathematics education in Quebec in 1900-1970 (Lavoie, 2003, pp. 201-317)
E. The language wars in Quebec: (Charland, 2005, pp. 174-175)
   a. Première Partie, Chapitre 2: L’échec de notre système d’enseignement, pp. 26-29
   b. Deuxième Partie, Chapitre 1: L’enseignement est-il une profession? pp. 68-71
G. Teachers’ Union or Teachers’ Professional Order? (Charland, 2005, pp. 180-188)
H. Quebec educational reforms in 1980s and 2000-10 (Charland, 2005, pp. 171-174)
I. Teachers’ competencies: (Martinet, Raymond, & Gauthier, 2001):43
   a. Professionalization of teaching: (Martinet, Raymond, & Gauthier, 2001, pp. 17-30)
   b. Core competencies of secondary school teachers of mathematics (Martinet, Raymond, & Gauthier, 2001, p. 165)
J. The present Quebec secondary school program, developed in 2001-2010:

42 All publications of Jean-Paul Desbiens are available at http://classiques.uqac.ca/contemporains/desbiens_jean_paul/desbiens_jean_paul.html
43 The MEQ 2001 Teacher Training document containing the list of Core Teachers’ competencies is available at: www.mels.gouv.qc.ca/dftps/interieur/pdf/formation_ens_a.pdf
44 MELS: secondary school cycle 1 program can be found at http://www.mels.gouv.qc.ca/dgfj/dp/programme_de_formation/secondaire/qepsecfirstcycle.htm
45 A pdf version of the mathematics secondary cycle 2 Quebec program is available at:
K. History of the General and Vocational Colleges or Collèges d’Enseignement Général et Professionnel” (CEGEP) in Quebec.\textsuperscript{46}

Questions on the readings

12. What was the contribution of Adrien Pouliot to mathematics education in Quebec? Hint: See (Lavoie, 2003, pp. 309-314), and a Wikipedia article on Pouliot.\textsuperscript{47}

13. What, if anything, was wrong with the education system in Quebec prior to the Quiet Revolution?

14. What changes to education in Quebec did the Parent Commission propose?

15. The formulation of teacher’s competencies 3, 4, 5 in (Martinet, Raymond, & Gauthier, 2001, p. 165) refers to student “competencies targeted in the program of study”. Find, in the present Quebec program of secondary study in cycles 1 and 2, the general competencies related to mathematics, and the specific competencies related to geometry.

Written Assignment 11: Designing a lesson

Design a secondary cycle 2 mathematics lesson whose main objective would be developing the competence of “Solving situational problems”, including all its five key features (shown on page 20 of the document “Mathematics – Second Version” which can be downloaded from http://www.mels.gouv.qc.ca/sections/programmeformation/secondaire2/index_en.asp?page=math)

References


\textsuperscript{46} Some information about cegeps, their history and mission, can be found at http://en.wikipedia.org/wiki/CEGEP in English, and http://www.lescegeps.com/documents/lohistoire_des_cegeps_du_sens_du_passe_a_celui_de_lavenir_par_jacques_dufresne in French.

\textsuperscript{47} Wikipedia: http://en.wikipedia.org/wiki/Adrien_Pouliot


