Construction of a Framework for Studying and Planning Elementary Mathematics Methods Courses

Helena P. Osana, Anna Sierpinska, Georgeana Bobos, and Klara Kelecsenyi
Concordia University

Osana, Sierpinska, et al. (2009)

- Mathematics teacher education
- Elementary mathematics methods (EMM) courses
- Objectives: To investigate how EMM courses are designed and run
- Participants: Instructors of EMM courses and their students (elementary preservice teachers) at 6 Canadian universities

Osana, Sierpinska, et al. (2009)

- Preliminary General Findings
  - No established or common resources for the mathematics teacher educator designing an EMM course
  - Instructors have varied backgrounds (training, education, experience)
  - Backgrounds govern their goals and approaches
  - No common framework for the design/study of EMM courses

Today's Presentation

- To propose an analytic framework (work in progress!) for the design and study of elementary mathematics methods courses
- To provide illustrations of how the framework can distinguish between two EMM courses
- Contributions
  - Creation of rubric for studying EMM courses
  - Important implications for teacher educators
  - Makes teacher educators' knowledge explicit, storable, communicable, and open to public examination (Hiebert, Gallimore, & Stiegler, 2002)

Literature Review: Grain of Description

1. Macro-level
   - Institutional organization of teacher education programs in various countries (e.g., Comiti & Ball, 1996; Stephens, 2003)
2. Micro-level
   - Epistemological and/or psychological focus on individual teacher (e.g., knowledge, beliefs, affect)
3. Meso-level
   - Level at which teacher educator operates
   - Plan the whole course, which is situated in a fixed, institutional context

Methodology

- Data (collected from instructors and students of EMM courses in three anglophone and three francophone universities in Canada)
  - Interviews (individual and focus groups)
  - Video- and audio-recordings of lectures and classroom activities (e.g., labs)
  - Document analysis (syllabi, assignments, tests, etc.)

Methodology

- Data Analysis
  - Grounded theory (Strauss & Corbin, 1998)
  - Open and axial coding of transcripts (interviews, lectures, classroom activities) and documents (assignments, syllabi)
  - Categories and themes emerged as components of the framework

Analytical Framework for Research on EMM Courses

I. Institutional Environment

II. Objectives
- A. Cognitive
  1. Mathematical Knowledge for Teaching (MKT)
     - a) Common Content Knowledge
     - b) Specialized Content Knowledge
     - c) Knowledge of Content and Students
     - d) Knowledge of Content and Teaching
     - e) Knowledge of Content and Curriculum
  2. Advanced Mathematical Knowledge (AMK)
     - a) Epistemology
     - b) Disciplinary Knowledge
  3. General Pedagogy
  4. Critical Reflection/Research Knowledge
- B. Affective
  1. Confidence
  2. Enjoyment
- C. Work Ethic
- D. Meta-level

III. Curriculum
- A. Topics
- B. Structuring Frameworks
- C. Tasks for Students

IV. Interactions with Students
- A. Form
- B. Format

Illustrations of 3 Dimensions of Framework

- Two Sites
  - Anglophone University 1 (AU1)
  - Anglophone University 2 (AU2)
- Illustrated Dimensions
  1. Institutional Environment
  2. Structuring Frameworks
  3. Formats of Interaction

1. Institutional Environment: Overview

AU1	AU2
Elementary teacher training (K – 6)	
120 credits over 4 years	
Two required EMM courses (6 credits)	
Teaching Mathematics I Teaching Mathematics II	Mathematics Teaching Mathematics
40 students per cohort	150-180 students per cohort
13 classes + review sessions	8 lectures + labs, workshops
Course website	
One TA for review sessions	Several TAs for labs and workshops
Review sessions planned	Labs planned, workshops not

1. Institutional Environment: Course Objectives

I. Institutional Environment

II. Objectives
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III. Curriculum
- A. Topics
- B. Structuring Frameworks
- C. Tasks for Students

IV. Interactions with Students
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- B. Format

2. Structuring Frameworks: AU1

- Examples
  - Cognitively Guided Instruction (Carpenter et al., 1999)
    - Children's thinking
    - Problem types (whole numbers and rational numbers)
  - Fundamental Fractions Concepts
  - Fundamental Concepts Related to Units (Measurement/Geometry)

2. Structuring Frameworks: AU1

Fundamental Fractions Concepts

1. Wholes can be divided into parts
2. Parts have to be the same size
3. Part is smaller than the whole
4. The size of the part is based on the size of the unit
5. Fractions are expressed in terms of the original unit
6. Parts can be combined to form wholes
7. Parts (fractional units) can be combined *no matter how many there are*
8. Each fraction has many equivalent representations

3. Formats of Interactions

- Bruner, J. (1985). The role of interaction formats in language acquisition. In J. Forgas (Ed.), *Language and social situations*. New York: Springer-Verlag.
- "Rule-bound microcosm in which the adult and the child do things to and with each other" …"the instrument of patterned human interaction"
- Determines boundaries and rules for what to say, how to say it, when to say it, and who says what in different circumstances
- Determined through explicit and direct negotiations between teacher and students (Sierpinksa, 1997)

3. Formats of Interactions

AU1	AU2
Teacher provokes cognitive conflicts	Teacher avoids cognitive conflicts
Teacher wants students to become aware of what they don't know	Teacher wants students to become aware of what they already know and/or can learn by themselves
Teacher uses structuring frameworks to help students resolve conflicts and learn more content/make connections	Teacher tells students to use intuition and everyday experience to resolve conflicts. In case of doubt, they are referred to resources.
Teacher provokes "why" questions and capitalizes on them to teach fundamental concepts of mathematical knowledge for teaching	Teacher dismisses "why" questions as "absurd" or reflecting lack of everyday experience

3. Formats of Interactions

AU1	AU2
Teacher's interactions with students follow the pattern: "make-a-mess-clean-it-up"	Teacher's interactions with students follow the "folk-constructivist" rule of "No telling"
Teacher leads students toward abstraction from real-life context of a particular problem	Teacher leads students toward more real-life experiences where mathematics can be used
Teacher designs tasks to mobilize elements of the structuring frameworks	Teacher chooses tasks according to (1) intuition, (2) some relationship to topic, (3) their capacity to illustrate the use of certain manipulatives or materials

3. Formats of Interactions: AU2

Setting

- Lab session on probability
- Students working in small groups on a lab assignment
- Materials required for assignment are available in the lab
- Solutions to the lab, sometimes with comments on students' solutions, posted on course website

3. Formats of Interactions: AU2

Task

Consider a deck of cards with only the 36 numbered playing cards (no aces, no face cards, no jokers). Evaluate the following game as fair or unfair and explain the difference.

You are dealt five cards and the dealer receives five cards. Your objective is to get the highest full house. In turn, you may discard one card and draw one card. The dealer will do the same, in turn. Do you each have the same chance of winning? If not, what is the difference? [A full house is three of a kind and a pair]

➔Task created because it has some relationship to the topic (i.e.., probability)

3. Formats of Interactions: AU2

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The game is fair (or that chances are the same) if the cards are dealt "normally" – that is one for me and one for you (and not five for me, five for you).

3. Formats of Interactions: AU2

Narrative/Utterance	Interpretation
In the lab, groups of students had long and heated discussions about this condition, and some never came to a consensus.	Conflict
The post for the above problem was: "Question 4. This strange card game that I invented is basically a fair game. However, I guess you could say that the person to take a turn first has the advantage of being first, particularly if it is only ONE event.	Uses intuition in an attempt to resolve conflict
It was obvious from discussing these card games with students that many have not had much card-playing experience. It's not too late. There are lots of card games that are just plain fun – playing cards can be a nice social activity for two people or for a larger group."	Uses everyday experience in an attempt to resolve conflict

3. Formats of Interactions: AU2

Narrative/Utterance	Interpretation
Someone asked: "Is there a justification for your assessment of the first game as 'basically fair'?"	"Why" question
The instructor's response was: "Your question seems a bit absurd. I don't think any justification is needed when I stated that the game is 'basically fair' – it is self-explanatory."	Dismissing as "absurd"

3. Formats of Interactions: AU1

Setting

- Second of 4 classes in a fractions unit
- Students working in small groups
- Unit designed around the theory of "progressive formalization" (Freudenthal, 1983; Romberg, 1998)
  - Students engage in solving problems
  - "Mathematize" their thinking, first intuitively, then being guided by teacher to "reinvent" their knowledge as it becomes more abstract and formal

➢ Plug: Osana and Royea (2010)
- CSSE 2010 Session 20.06 (Tuesday, 10:00 a.m.)

3. Formats of Interactions: AU1

Task

1. Work together in your small groups to solve each of the problems in the card set (there are 18 all together).
2. For each problem, find a solution by drawing pictures to represent the amounts in the problem and the actions you need to perform.

➔Task designed to mobilize elements of structuring framework (fundamental fractions concepts)

3. Formats of Interactions: AU1

- Students are working on the problem:

There are 12 pies left over at the end of a busy day at the bakery. The owner says you can only take 2/3 of them home. How many pies are you allowed to take home?

3. Formats of Interactions: AU1

- Students are working on the problem:

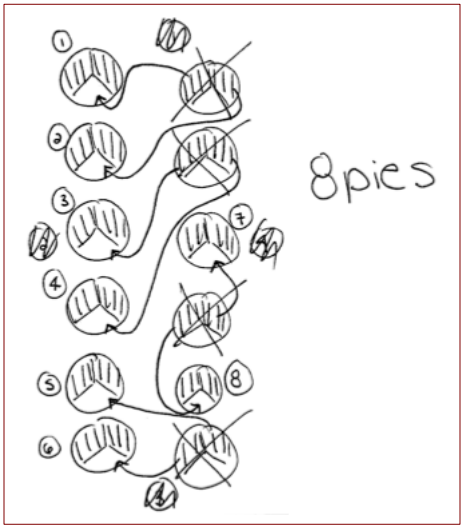
> There are 12 pies left over at the end of a busy day at the bakery. The owner says you can only take 2/3 of them home. How many pies are you allowed to take home?

$$\frac{2}{3}(12\,pies) = 8\,pies$$

3. Formats of Interactions: AU1

Conflicts during interaction

- Constructing a concrete representation (2/3 of 12)



3. Formats of Interactions: AU1

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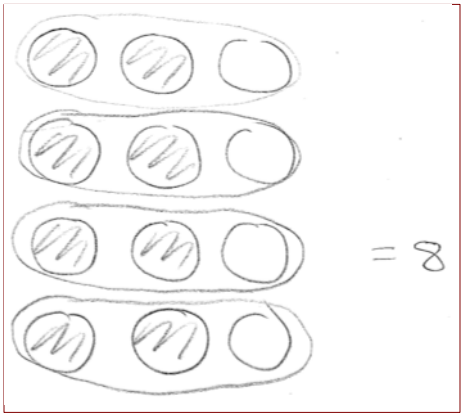

3. Formats of Interactions: AU1

Other conflicts

- Identification of the whole (1 pie? 12 pies?)
- Representation of final answer (8/12 pies? 8 pies?)
- Meaning of fractions in relation to the whole
  - Is 8/12 of one pie the same as 8/12 of 12 pies?

3. Formats of Interactions: AU1

Ln		Utterance	Interpretation
29	S2:	Cause I get confused when it's two thirds of a pie or two thirds of all the pies. Like let's say…	Starts expressing a conflict
30	AU1:	Mmm.	
31	S2:	Someone eating two thirds of a pie.	Making conflict more explicit
32	AU1:	Right	
33	S2:	That's of one pie. If they ate two thirds of them, that means two thirds of all the pies.	Making conflict even more explicit
34	AU1:	So in this case what is your whole, in this case?	Pointing to a piece of a structuring framework that may resolve the conflict
35	S2:	Our whole is three. Isn't that our unit? Like… I don't know…	Trying to use the piece of framework, but doesn't work.
36		[Students talking, incomprehensible]	
37	AU1:	Ok. Why do you think the whole would be twelve?	
38	S3:	Cause there's twelve pies.	
39	AU1:	Cause there are twelve pies? Ok. And why is it not one pie?	Attempting to test understanding
40	S3:	Cause you want two thirds of twelve pies.	
41	AU1:	Right. Right. Yes, the whole is twelve in this case…	

3. Formats of Interactions: AU1

Ln		Utterance	Interpretation
42	S4:	You …break these others into [incomprehensible]	
43	AU1:	…and so you're taking two thirds of the whole.	Reiterates part of the framework
44	S1:	Yeah.	
45	S2:	Two thirds of the whole. Just twelve.	
46	AU1:	That's right.	
47	S2:	but if they were to say two thirds of a pie [those to the whole?] would mean that one pie.	
48	AU1:	Right.	
49	S2:	That would… Ok, that would be our whole.	
50	.		
	.		
66	AU1:	But eight out of twelve…	
67	S2:	Is two thirds of twelve pies. No, eight is…Yeah, yeah, yeah! Eight is two thirds of twelve.	
68	AU1:	Ok, so show me how you got those numbers. What did you do to solve that problem?	
69	.		
	.		

3. Formats of Interactions: AU1

Ln	Utterance	Interpretation
78	AU1: So ok, so then, so then eight out of twelve. The way you expressed that there, it's a fraction. Is it, is it bigger than one or less than one? Eight out of twelve?	
79	S2: Less than one! Cause twelve out of twelve is one.	
80	AU1: That's right! So it's less than one but you're telling me at the same time you can take eight pies home. So how can you take eight out of twelve? How can you take eight out of twelve pies home?	Provoking conflict
81	S1: Yeah, if twelve is not the whole…	
82	AU1: What is twelve in this case? If you're looking at the fraction eight over twelve what is twelve?	
83	S1: Twelve is, wait…	
84	AU1: If you're looking at the fraction eight over twelve?	
85	[Students talking, incomprehensible]	
86	S1: Eight pies out of twelve.	
87	AU1: Forget the pies. If you're looking at eight over twelve.	Leading students toward abstraction from real-life context
88	S1: Oh, eight pieces out of twelve pieces, let's say.	
89	AU1: Right. Of what? Twelve pieces of what?	
90	S1: Of one whole.	

3. Formats of Interactions: AU1

Ln	Utterance	Interpretation
91	AU1: Of one whole. Right. So basically you're looking at something that is divided into twelve parts.	Pointing to another piece of the framework
92	S1: And each part [incomprehensible].	
93	AU1: Right. Is this …	
94	S1: That's what this is saying	
95	AU1: Is this that's what that is saying? Eight out of twelve…	
96	. .	
119	S3: Because that would mean that the pie is cut into twelve pieces and you're taking eight out of twelve.	
120	S4: No, no, no. You're not doing that.	
121	AU1: No, you're not doing that. Well, wait a minute. How do you…	
122	S1: If it…That's wrong.	
123	S3: Nooo, but it's not necessarily wrong, because you're still saying… it depends how you present the fraction.	Stating a piece of the framework
124	AU1: It does depend how you represent the fraction.	Repeating the piece of the framework – originally given by the student

3. Formats of Interactions: AU1

Ln		Utterance	Interpretation
125	S3:	Because if you are representing the fraction as each pie… I don't know how but it doesn't… and it makes sense at the same time.	
126	S2:	Eight whole pies. Yes, that's how much this person can take.	
127	AU1:	So the answer is…	
128	S2:	Eight whole pies.	
129	AU1:	So write that down eight whole pies.	Highlighting the correct solutions that come from the students
130		[Students' voices overlapping] Eight full pies.	
131	AU1:	It is right. Eight whole pies.	Highlighting the correct solutions that come from the students
132	S2:	Eight whole pies out of twelve whole pies, not eight out of twelve.	
133	AU1:	Right. It is how you represented it.	Repeating part of framework
134	S1:	How I represented it?	
135	AU1:	It's not eight out of twelve.	
136	S1:	It's not eight out of twelve. One pie…	
137		[Students talking, incomprehensible]	
138	AU1:	Eight out of twelve is a quantity. This is a quantity.	Pointing to another piece of framework

3. Formats of Interactions: AU1

Ln		Utterance	Interpretation
139	S1:	Oooooh! You're asking for…Ok, ok, ok, ok.	
140	AU1:	And this quantity is less than one. But it can't be less than one [pie]; you're taking eight whole pies home.	
141	S3:	Now I see the problem because every… this is less than one.	Conflict resolved
142	S2:	A fraction.	
143	S3:	And a pie is …	
144	AU1:	Is more. And eight pies is more than one pie.	
145	S3:	That's right [laughing]. It makes sense…	

Summary

- Analytical framework for characterizing and distinguishing EMM courses
- Framework is sensitive to features of the course and instructor's goals, approach, and knowledge
- Research ongoing: complete analysis of all sites is underway

Conclusions

- Institutional environments constrain several dimensions of the analytical framework
- In particular, the institution frames the formats of interaction that cannot easily be transplanted to other contexts
- Vast knowledge of both mathematics and mathematics education is required to negotiate fruitful formats of interactions with prospective teachers

Conclusions

- Teaching future elementary teachers of mathematics is more complex than training secondary school teachers
  - Problems/obstacles occur at a more fundamental and philosophical levels
  - It takes a specialized type of knowledge to be an effective mathematics teacher educator

Helena P. Osana
Department of Education

Anna Sierpinska
Georgeana Bobos
Klara Kelecsenyi
Department of Mathematics and Statistics

Concordia University


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