LECTURE 8

DISCOVERY TEACHING/LEARNING AND THE THEORY OF DIDACTIC SITUATIONS.

1. HOW 'GOOD' OR 'BAD' IS THE THEORY OF SITUATIONS?

IS THIS A GOOD QUESTION TO ASK ABOUT THE THEORY?

Lessons or lesson plans cannot be said to 'conform' or 'not conform' with the theory of situations or didactic engineering. The theory of situations provides us with a perspective from which to better understand what is going on in the classroom in terms of the students' learning, the mathematical meanings that the students construct, and the role, in this process, of the teacher.

When we design a lesson with the theory of situations as a guide then we are not trying to conform to some set of rules or norms but we are trying to foresee the effects of our decisions and choices with the tools of the theory. Thus, we may come to the conclusion, that in organizing the classroom activities in such and such way, all we'll obtain is an expected behavior in the students (for example, they will produce an expected equation) but not the expected understanding. Then we may try to change the lesson plan so that the expected understanding does develop in the students. However, no matter how hard we try, we may still be wrong in our expectations, and when experimented, the lesson will not produce the understanding and knowledge we aimed at. If this happens, it is not the theory that is to blame but our design in which we may have failed to take into account certain variables.

Here is what Brousseau wrote in respect to the status of the theory of didactic situations in an e-mail message sent to me in the last week of November 1999.

> 'The theory of situations is aimed to serve both the study and the creation of all kinds of learning and teaching situations, whether they are "spontaneous", or the product of an experience or of a special engineering project, and whether they are efficient or not. It is not a method of teaching. The theory can provide some methods of teaching, it can justify some methods and disqualify some other methods, as the case may be. The theory contains models that may support certain plans of action aiming at making the students (re)discover some mathematics. This way the theory can make suggestions for engineering. The didactic situations of "rediscovery" can thus be, in general, linked to these models' (my translation).

The practical significance of a didactic theory can be defined, partly, by its relationship with a methodology of instructional design. In the case of the theory of didactic situations, the corresponding methodology is that of didactic engineering. This is how Brousseau explains the relationship between the two:

'Theory of situations has been elaborated to provide a framework for the study and means of description of any situation in which there is an intention of teaching someone some precise knowledge, whether it succeeds in it or not. It can take into account all the forms of learning identified in all kinds of research. The theory does not pretend to present *all* aspects of teaching situations and replace *all* the approaches: psychological, psychoanalytical, linguistic, statistical, etc.. But it tends to put the contribution of these approaches in the perspective of their function and their generality in the description of "didactic phenomena". One could use here an analogy with economics and its relation with commerce. Economics does not deal with the psychology of the vendor or of the buyer but it can take into account their impact on the macroeconomical phenomena.

Theory of situations is neither an ideology nor particular didactic method. In this sense, it has no technical alternative. It does not directly recommend this or that particular didactic procedure. Its theoretical concepts only allow one, for reasons of consistency, to predict the role of certain factors in some circumstances. This way it puts limitations on what it is possible to do or change in teaching, just as thermodynamics discards the possibility of building a *perpetuum mobile* but does not give precise guidelines for the construction of an ideal engine.

On the other hand the theory manages well the contingency or the possibility that some of its theoretical predictions be rejected as false. For example, at one time I claimed that "a situation of formulation related to a precise piece of knowledge cannot function if a previous situation of action has not allowed the students to develop an implicit model of this knowledge". But, later, I found counter-examples and I had to restrict the generality of the claim that, previously, made a lot of sense to me. Another example: I claimed that "An implicit model of action that is not, rather quickly, supported by a formulation, is lost as fast as it is learned". I had to soften this declaration by provisions such as "in most cases", "if the situation of action is evolving", etc..

In this sense, the relevance of the theory lies in its ability to raise questions and classify and order the answers. (One of the difficulties of mathematics education is the multiplicity and diversity of the research work, classified according to criteria which appear to have nothing to do with each other. The important thing is thus to be able to judge of the relevance of the questions and the validity of answers).

Yet, if I have developed the theory of situations it is because I needed it for didactic engineering. It is a product of my efforts to classify questions that are raised when one wants to organize, by whatever means, the learning, by anyone, of some knowledge. This explains why one finds, in the description of the types of situations, arrangements corresponding to the possibility of the construction of knwledge by the students in a non didactic situation. This is what resembles most, it seems, to the settings in which one can detect the "discoveries" of the students and imagine their role in the learning processes. There remains the question of interpretation, in the theory of situations, of the conclusions and claims of the observers of "discovery learning".

This way, the theory of situations restricts the meaning that one can attach to "didactic engineering". The point is to distinguish between didactic "innovations" or "inventions" made for the educational market, and those products of the didactic work that are based on proved techniques, described by explicit and sensible technologies, which are, in turn, founded on falsifiable theories and verifiable experiences. The distinction is not easy because, in any work of didactic engineering, inventions play a very important part, as do empirical considerations which can be extremely complex and possibly judicious. Moreover the success of these products at differents levels (students, teachers, society) is not decisively determined by their belonging to this or that category.

I am presently trying to fill the gap that I have left between the theory which presents the wide basic concepts and general didactic phenomena, and the different products of the didactics proper (the teaching of natural numbers, decimal numbers, rationnal numbers, measure, statistics and probability, reasoning and logic, space and geometry, elementary arithmetic and algebra) whose practice I have organized and observed. I am dreaming of a big treatise on didactic engineering which I consider as the didactics proper.' (My translation)

DISCOVERY LEARNING APPROACH

I have judged one of the lessons presented in the class by students (related to Pythagoras theorem) as an example of the 'discovery learning' approach. I did not mean to say that it is not 'conform with didactic engineering'. I only meant to say that, if one analyzed this lesson from the theory of situations perspective prior to actually conducting the lesson then one would either change the plan of the lesson or interpret the outcome of the lesson differently: the evaluation of what the students have actually learned in that lesson would be different.

What I understand by 'discovery learning approach' is an organization of the students' activity in a sequence of exercises so that, at the end, some representation of the target knowledge is produced by the students (e.g. a formula is written, a term is said, a drawing is produced), so that, in the eyes of one knowing the target knowledge, the manifestation of this knowledge in the lesson is obvious. In the approach presented by one of you, in the activity phase, the students were already manipulating representations of the target knowledge; the formulation phase was strongly guided by the teacher (Topaze effect), and the validation phase occurred after the 'discovery' (Jourdain effect): the students were then asked to prove a theorem which they already knew to be true.

I asked Brousseau, by e-mail, for an explanation of the relations between the discovery learning approach and the theory of situations and he kindly responded (last week of November 1999, same message as above). I partly translate and partly summarize, below, his answer.

'The theory of situations makes it possible to identify the circumstances of the discovery by the students (or anybody, for that matter) of some knowledge but, within the theory, the notion of "learning through discovery" appears contradictory. On the other hand, the theory can recognise the notion of "teaching through discovery" as referring to the organization of non didactic situations, and to the identification, the interpretation and the institutionalization of the knowledge produced by the students in these situations. The "teaching through discovery" approach creates situations based on "an epistemological and didactic fiction" which favors an auto-didactic reading of these situations by the students.

Briefly speaking, the "discovery learning" appears to be a myth, a psychological extrapolation invented to justify and conceal a "discovery teaching": a method of rediscovery. The problem solving method is based on heuristic interpretations of the mathematical activity, as opposed to the classical axiomatic and purely deductive interpretations. It leads to epistemological commentaries and "original" terminology and methods. Heuristics start to be taught as ordinary objects of teaching. These heuristics are conjectures about the learning of mathematics that are false in all their generality but useful or valid in an important number of cases. But they are indeterminate, otherwise they would become theorems. There is no reason why these heuristics would work better than the "real theorems"; in that case one could look for second order heuristics to produce and guide the first order heuristics. There is no doubt that heuristics, like, for example, analogies, are useful for the researcher who engages them under his or her own responsibility. But when they are proposed to the students as methods, in the frame of a didactic contract, they are nothing but some of the invisible means of the teacher to conceal his or her didactic strategies and produce the Topaze or Jourdain effects.

This being said, there is no reason why heuristics should not be used when they work. The theory of situations does not make a mystery out of the theatrical component of any didactic situation. To play at discovery is not as realistic as to play at a competition of theorems but it can be as exciting and maybe more flattering, more gratifying for the ego of the student as well as of the teacher; briefly, it can be, locally, just as efficient for the student's learning.

From a strictly scientific point of view, it is not necessary to invent false psychological and cultural mechanisms to justify this didactic approach. But in pre-service teacher training, if didactic situations are presented for what they are, then perhaps the future teachers will not fall for it. It might be that they need to "believe in the reality of movie characters". To be successful in teaching one needs such a will or conviction that perhaps the easiest thing to do would still be to justify this professional "faith" by appropriate beliefs?' (my translation)

It seems to me that the debate should focus on three points:

- The relations between the theory of situations and the didactic engineering;

- The relations between the 'discovery learning' approach and the theory of situations;

- The possible difference between the meaning of the word "theory" in Educational Sciences, especially anglo-saxon, and its interpretation in natural sciences, and the difference of perspective on the nature of didactic phenomena where the make believe and the masquerade are the standard practice.