Seed of an index of categories for analyzing and planning "Teaching Mathematics" courses for prospective elementary school teachers

Supporting documentation for reports of research on elementary mathematics methods courses, conducted in the years 2008-2011 by Anna Sierpinska and Helena Osana from Concordia University, Montreal, QC, Canada. The research was supported by a Social Sciences and Humanities Research Council of Canada grant no. 410-2008-28981.

Introduction

In our research, we sought means for describing the content of university courses aimed at preparing prospective elementary school teachers for teaching mathematics. Such courses used to be called "Elementary Mathematics Methods" in English language universities and they were assumed to introduce future teachers to methods of teaching mathematics. They have been replaced by courses with names where the word "methods" does not appear. We will call them "Teaching Mathematics" courses and use the abbreviation "TM". If "methods" are not taught in the courses, then what the future teachers are offered instead?

In our research, we visited 6 Canadian universities in 3 provinces. As a result of our research we propose not so much a definite answer to the question posed above for the TM courses we visited, as a way to develop a tool to address this question with respect to any TM course. This tool is a system of categories of ACTIONS in which the courses engage future teachers, combined with a system of categories of ANALYTIC TOOLS offered to the future teachers to guide them in performing the actions. We think of the two systems as a seed of an expandable "indexing framework" (Hiebert, Gallimore, & Stigler, 2002, p. 8) for TM instructors, in a similar way that curriculum is an indexing framework for mathematics teachers. We think that having a common indexing framework for sharing information and reflection about the TM

courses could be an important step towards the professionalization of TM instructors' knowledge. The framework would be "expandable" in the sense that categories could be added to it without destroying the existing ones. Researchers and instructors could add categories to the index by analysing different TM courses, and make their additions public, thus contributing to the growth of a shareable, collective, knowledge base. We are thus taking Hiebert et al.'s dream of a "professional knowledge base for mathematics teachers" to the level of mathematics teacher educators.

In this document we present a seed of the expandable index, obtained based on analyses of two TM courses, taking into account several implementations of these courses in the years 1999-2010.

Abbreviated indexing framework for analyzing and planning Teaching Mathematics courses

I. ACTIONS that future teachers are expected to perform in a Teaching Mathematics course

- I.1 Teacher-to-teacher professional communication
- I.2 Teacher's action
  - I.2.1 Teaching activity preparation
  - I.2.2 Teaching
  - I.2.3 Reflection on teaching
  - I.2.4 Assessment of students' work
- I.3 Student's action
  - I.3.1 Behaviour
  - I.3.2 Didactic-theoretical action
  - I.3.3 Mathematical action
- I.4 Future teacher's action

II. ANALYTIC TOOLS suggested by TM instructors for carrying out the actions

- II.1 TK – Models of teacher's knowledge
- II.2 TDA – Models of teacher's didactic actions
  - II.2.1 Teaching models
  - II.2.2 Teaching techniques
  - II.2.3 Teaching principles
  - II.2.4 Phases of the teaching act
  - II.2.5 Lesson types
  - II.2.6 Materials for teaching
  - II.2.7 Models of assessment
- II.3 MKT – Models of mathematical knowledge for teaching
  - II.3.1 Curriculum
  - II.3.2 Problem types
  - II.3.3 Elementary school mathematics
- II.4 PML – Psychology of mathematics learning
- II.5 Comm – Models of communication skills
- II.6 Prof – Models of professional behavior

Seed of an indexing framework for analyzing and planning Teaching Mathematics courses

ACTIONS *that future teachers are expected to perform in a Teaching Mathematics course*			
Teacher-to-teacher professional communication			
	Curriculum outcomes		
		interpretation	
			Producing an html document explaining the meaning of a curriculum outcome, with hyperlinks to other html documents on related outcomes
		interpretation critique	
	Resources for teaching		
		sharing	
			Referencing; Indexing by curriculum outcomes; Indexing by process standards for mathematics; Abstract writing; Communicating ideas about using resources; After a presentation of resource-using ideas to an audience, distributing a lesson plan that illustrates these ideas
		publishing	
			Creating a webpage about resources
		critiquing	
			Critique of the resource's appropriateness as a reference; Critique of the resource's presentation
	Reflection		
		shared with fellow teachers, on…	
			own teaching, triggered by feedback from peers; fellow teacher's teaching; an issue; assessing mathematical understanding; standards of the teaching profession

Teacher's action			
	Teaching activity preparation		
		Year plan writing	
		Unit plan writing	
		Lesson plan writing	
		Materials development	
			Handouts or notes for distribution in class; Teacher's Guide writing
		Homework preparing	
			Math Bag creating (Reid, 1999)
		Writing a problem satisfying given conditions	
			Traditional mathematical story problem; A realistic word problem; Open-ended problem; A problem requiring a given interpretation of multiplication; A division problem where remainder must be ignored / considered; A multiplication/division situational problem involving multiplication/division sub-problems other than those in a given problem
		Proposing modifications to a lesson viewed on a video	
			With the goal of provoking students to use more advanced problem solving strategies
		Predicting students' mathematical behavior	
			Predicting: how a student would use a certain type of strategy in solving a problem; how a student observed to use a certain type of strategy in solving a problem would solve another problem; students' errors on a type of calculation task;

			students' strategies in solving a given problem; students' strategies in solving a given type of problem knowing that he/she does not possess a given problem solving skill
		Assessing the relative difficulty of two or more given problems	
	Teaching		
		Simulation of teaching	
			With fellow future teachers in the role of students
		Authentic teaching	
			As instructor of a TM course
		Imagined teaching	
			Describing how to explain a mathematical concept to children, e.g. the concept of place value, or the standard division algorithm, using concrete materials
	Reflection on teaching		
		Not shared with fellow teachers	
			On: own teaching; fellow teacher's teaching; a video of teaching; an issue; the meaning of a mathematical concept, e.g. equality sign; students' conceptions of a mathematical concept; Lesson plan critique
	Assessment of students' work		
			Deciding if a student's solution is correct; Finding a student's error in a calculation; Identifying mastered and not yet mastered skills, implicit in a student's solution of a counting problem; Assessing student's understanding of subtraction based on his/her solution of a single calculation

Student's action			
	Behavioral		
		Class participation	
		Course outline reading	
	Didactic-theoretical		
		Identify a didactic-theoretical object (recognize and name)	
			type of problem; category of a problem-solving strategy; a concept-in-action in a child's solution; a teaching technique used by a teacher in a video; the teaching objective of a teaching activity
		Define a didactic-theoretical object	
			of the Cognitively Guided Instruction, of the difference between a problem and an exercise; of the expression "place value"
		Classify	
			a set of traditional word problems into types
		Model	
			relationships in a word problem using various material and symbolic representations
		Reason	
			within a theory of mathematics learning: draw conclusions from principles formulated in the theory; refute a common belief about mathematics learning using arguments from the theory (e.g. "Only kids who are bad at math need to count on their fingers to compute"); apply the definition of a problem solving strategy to show a possible solution of a problem
		Answer questions about a reading on teaching theory	
			(Ball, Hill, & Bass, 2005)

	Mathema-tical		
		Solving an elementary math word problem	
			Problem of the week (Drexel University, 2011); A complex / simple situational problem; A typical school word problem
		Discussing solutions to an elementary math word problem	
		Solving problems about representations of numbers	
			Counting by grouping and re-grouping material objects; Counting in abstraction, i.e. saying or writing consecutive numbers forward or backward, in base ten or other than ten; Converting one representation of a number into another (verbal, formal, material, pictorial, base ten, base other than ten); Finding a general relationship between the length of representations in different bases; Deducing the base of the place-value system from a number sentence; Deciding if a formal representation of a number is correct; Ordering numbers written in base other than ten Rounding off
		Solving problems about arithmetic operations	
			Using standard algorithms for arithmetic operations in base other than ten; Converting a representation of an arithmetic operation into another representation (formal, pictorial, material, base ten, base other than ten); Performing an arithmetic operation using a variety of algorithms (Standard algorithm,

			Base-10 blocks; A non-proportional model; Expanded form for addition; Partial sums algorithm for addition; Partial products algorithm for multiplication; Equal addition algorithm for subtraction; Using a grid to calculate a product of two whole numbers; Multiplication based on distributive property); Explaining the relationships between pairs of algorithms for arithmetic operations; Identifying properties of arithmetic operations implicitly used in a transformation of an arithmetic expression
		Defining mathematical concepts	
			Equals sign; Even numbers; Place value
		Proving mathematical statements: deciding if a statement is true and justifying one's answer	
			If 0 is even or odd; if subtraction is commutative; if division is commutative; if whole numbers are closed under subtraction; if a technique employed in a single problem is generalizable to all problems of the same type;
Future teacher's action			
	Reflection about the future profession		
		Describing the model elementary math teacher	
		Critiquing other people's views of the model elementary math teacher	

	FTs' communal decision making		
		Deciding on topics to study in a TM course	
		Deciding on assignments in a TM course	
	FT's individual decision making		
		Deciding on the grading scheme to be applied to one's own work	
ANALYTIC TOOLS ***suggested by TM instructors for performing the actions***			
TK ***Models of teacher's knowledge***			
	Mathemati-cal Knowledge for Teaching (Ball, Hill, & Bass, 2005)		
TDA ***Models of teacher's didactic actions***			
	Teaching models		
		Traditional teaching	

		Interactive teaching	
		Problem-based teaching	
			Structure of a problem-based lesson \ the Before, During and After phases; (Van de Walle & Lovin, 2006)
		Action-Representation-Symbol-Formalism (ARSF) model	ARSF = progression from actions on concrete objects, through representations, including formal operations if applicable
		Mathematics Learning Environment (MLE)	
			Dimensions of an MLE \ The nature of classroom tasks; The role of the teacher; The social culture of the classroom; Mathematical tools as learning supports; Equity and accessibility (Hiebert, et al., 1997)
	Teaching techniques		
		Flexible Interviewing Techniques	Basic structure of a flexible interview: 5 steps \ Present students with a task; Check that they understand the task; Investigate their thoughts and interpret their responses; Draw a conclusion about what the child knows or understands; Teach during or after the interview (Ginsburg, Jacobs, & Lopez, 1998)
	Teaching principles		
		NCTM Principles and Standards (National Council of Teachers of Mathematics, 2000)	Process Standards \ Reasoning and proof; Problem solving; Communication; Connection; Representation Content Standards Principles for school mathematics \ Principles of Equity; Curriculum;

			Teaching; Learning; Assessment; Technology
		Characteristics of an effective MLE	Classroom tasks should be genuine problems; Teacher's primary responsibility is to establish a classroom culture in which students choose and share their own methods for solving problems and where the correctness of an argument resides in the mathematics and not in any one individual; Teacher encourages students to use tools for solving problems and to construct their own tools for the purposes of recording; All students contribute to the discussion and are heard by their peers thus ensuring that all have access to the mathematics being learned (Hiebert, et al., 1997)
		Basic principles of flexible interviewing	
			Be responsive to students; Ask questions that are non-directive; Use child's own words; Ask questions regardless of whether the child is right or wrong; If the task does not work or the child is losing interest, present it in different way or with different manipulatives (Ginsburg, Jacobs, & Lopez, 1998)
		Appropriate assessment	
			Assessment aligned with teaching goals and teaching model Assessment that allows students to demonstrate what they know, and for which students have no prescribed solution technique are most appropriate for problem-based teaching (Van de Walle & Lovin, 2006, p. 29)
		Resource use standards	
			Practicality; Connections with curriculum; Connections with learning theory

	Phases of the teaching act		
		Pre-teaching activities	
			Pre-planning considerations about curriculum, students' learning, goals, types of activities; Planning \Year plan; Unit plan; Lesson plan
		Teaching	
			Choosing a teaching model
		Post-teaching activities	
			Evaluation; Self-reflection questions (Cathcart, Pothier, Vance, & Bezuk, 2000)
	Lesson types		
		Introductory lesson; Application lesson; Review lesson	
	Materials for teaching		
		Formats of materials for teaching	
			Year plan template; Unit plan template; Lesson plan template; Teacher's Guide; Math Bag; Webpage; Resource list; Handouts; Worksheets; Questionnaires
		Standard content elements for a teaching material	
			Key ideas (aims, goals, objectives), and their (justified) and their *Connections* with curriculum outcomes; Classification of the curriculum outcomes into those that have to be attained in Prior learning, Concurrent learning, and Subsequent learning; Connections with other subjects

			(multidisciplinarity); *List of objects* to be brought by teacher or students; *Procedure* (description, activities, problems); *Expected answers or solutions*; *Suggestions for evaluation* (assessment); *Homework* (at-home learning activity) *References* and Reference components
		Standards of quality for teaching materials	Clear planning logic; Structure appropriate for a the chosen type of teaching method; Key ideas (curriculum focus) appropriate for the given grade level; Conceptual relations are made between curriculum outcomes and the key ideas for the activity; Multidisciplinary connections are plausible; Consideration of NCTM content, process standards; Progression from actions on concrete objects modeling concepts to more abstract representations; Use of manipulatives; Awareness of constructivist learning theory; Level of student thinking encouraged; Level of student involvement encouraged; Consideration of equity and diversity issues; Support for special needs students (strong/weak background, vision, hearing, mobility problems, language of instruction different from student's mother tongue); Use of appropriate assessment; Suitability of the at-home components to use by parents; Practicality; Correct spelling, punctuation and word use; References follow referencing conventions; References are indexed by curriculum outcomes and/or NCTM process standards

	Models of assessment		
		Assessment types	
			Recall assessment; Skill assessment; Close-response items assessment; Performance assessment; Diagnostic interview (Van de Walle & Lovin, 2006, pp. 29-36)
		Assessment tools	
			Grading; Rubrics (ibid.) (Van de Walle & Lovin, 2006, pp. 29-36)
MKT ***Models of mathematical knowledge for teaching***			
	Curriculum		
		Topics	
			Number sense; Operations; Patterns and relationships; Measurement; Geometry; Data; Probability
		Outcomes	
			Grade Curriculum Outcomes; Specific Curriculum Outcomes
	Problem types		
		Exercises	
		Genuine problems	
		Traditional mathematical story problem	
		Open-ended problem	
		Realistic word problem	
		Situational problem	
		Problem types according to operations	
			Division problems where remainder must be ignored / considered

			<u>Types of addition and subtraction problems according to</u> \ Join problems; Separate problems; Part-Part-Whole problems; Compare problems (Carpenter, Empson, Fennema, Franke, & Levi, 1999); <u>Types of multiplication/division problems</u> \ *Asymmetric problems* (including Rate, Price and Multiplicative Comparison) \ Multiplication problems; Measurement division problems; Partitive division problems \ *Symmetric problems* \ Area problems; Array problems; Combination problems (Carpenter, Empson, Fennema, Franke, & Levi, 1999)
		Problem types according to arithmetic sentences	
			Division sentence; Missing factor multiplication sentence
	Elementary school mathematics		
		Mathematical Principles of Counting according to (Ginsburg H. , 1989)	
			"Anything can be counted"; "The number sequence is endless"; "Each member of the set must be counted once and only once" (One-to-one Principle); "Match up each number word with each thing; each object must be assigned one and only one number word (Uniqueness Principle)"; "Say the number words in their proper order (Stable Order Principle)"; "The things counted need not look alike [there are no special numbers for counting

			rocks and for counting chairs]"; "The order of counting does not matter"; "The physical properties of members of a set do not matter"; "Counting tells us how many objects there are in the collection as a whole (Cardinality principle)".
		Numeration systems	
			Place-value systems \ notion of base; notion of place value; base-four numerals; base-ten numerals; grouping; regrouping; base-four "categories" (positions)
		Arithmetic operations - algorithms	
			<u>Standard algorithms</u> \ With numerals written in base other than ten \ Formal representation; Pictorial representation \ With numerals written in base ten \ Formal representation; Material representation \ Addition and subtraction \ Multiplication and division <u>Non-standard algorithms</u> \ Base-10 blocks; A non-proportional model; Expanded form for addition; Partial sums algorithm for addition; Partial products algorithm for multiplication; Equal addition algorithm for subtraction; The grid model for multiplication; Multiplication using the distributive property <u>Relations</u> between different algorithms;
		Properties of arithmetic operations	
			Commutativity; Associativity; Distributivity Closure of a number set under a given operation; Divisibility relationships in integers

		Even numbers	Informal definitions for different grade levels; Formal number-theoretical definition
		Equals sign	
PML ***Psychology of mathematics learning***			
	Learning theories		
			The van Hiele model Constructivism \ according to Piaget; Skemp; Dienes Theoretical underpinnings of the Math Bag program
	Learning-teaching theories		
		CGI	Cognitively Guided Instruction according to Carpenter et al. (Carpenter, Empson, Fennema, Franke, & Levi, 1999)
		Psychology of learning to count according to Ginsburg (1989)	
			Children's counting strategies; children's counting errors
		Problem solving heuristics	
			Problem-solving "strategies" inspired by Polyá's "How to solve it"
		Children's problem-solving strategies	
			According to CGI (Carpenter, Empson, Fennema, Franke, & Levi, 1999) \ Direct modeling; Counting; Using derived number facts; Grouping; Measurement
Comm ***Models of communication skills***			
	Presentation standards		
			Clarity; Creativity; Scope; Depth
	Discussion		

	standards		
			Insightful ; Creative; Thought-provoking; Timely; Adding new discussion topics; Generalizing from the particular; Making the general specific; Making connections; Offering interesting speculation
Prof ***Models of professional behavior***			
	Course participation standards		
			Attendance; Contributions to class discussions; Assisting peers in collaborative activities
	Standards of student behavior		
			Course outline reading; Textbook and other texts reading; Assignments and their elements understanding; Formatting guidelines for assignments following; Due dates respecting; Arriving in class on time, staying until dismissed; Maintaining positive attitude towards learning the content of the course
	Standards of the teaching profession		
			Being prepared for class; Participating constructively; Attending classes; Attending to class activities; Compensating for absence
	Professional behavior		
		Mechanisms of accounting for professional behavior	

Bibliography

Ball, D., Hill, H., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(1), 14-17, 21-22, 43-46.

Burris, A. (2005). *Understanding the math you teach: Content and methods for prekindergarten through grade 4.* Upper Saddle River, NJ: Pearson.

Carpenter, T. P., Empson, S. B., Fennema, E., Franke, M. L., & Levi, L. (1999). *Children's mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

Cathcart, G., Pothier, Y., Vance, J., & Bezuk, N. (2000). *Learning mathematics in elementary and middle schools. Third edition.* Toronto: Pearson.

Drexel University. (2011). *The Problems of the Week Program*. Retrieved October 28, 2011, from The Math Forum @ Drexel. Peoplelearning math together: http://www.mathforum.org/problems_puzzles_landing.html

Ginsburg, H. (1989). *Children's arithmetic: How they learn it and how you teach it.* Austin, TX: Pro-Ed.

Ginsburg, H., Jacobs, S. F., & Lopez, L. S. (1998). *The teacher's guide to flexible interviewing in the classroom: Learning what children know about math.* Don Mills, ONT: Allyn & Bacon.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., et al. (1997). *Making sense: teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one? *Educational Researcher, 31*(5), 3-15.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Reid, D. (1999). *Math bags*. Retrieved May 23, 2011, from Acadia University: http://plato.acadiau.ca/courses/educ/reid/4173/Mathbag/index.html

Reys, R., Lindquist, M., Lambdin, D. V., Smith, N. L. , & Colgan, L. E. (2010). *Helping children learn mathematics. Canadian edition.* Toronto: John Wiley & Sons Canada, Ltd.

Van de Walle, J. A., & Lovin, L. H. (2006). *Teaching student-centered mathematics: Grades 5-8.* Pearson Ally & Bacon.